ARITHMETIC PERCOLATION

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ABSTRACT. Plant a thin tree at each vertex of the square lattice. Then, pick a tree uniformly at random in a huge box, and replace it with a lamp. A tree is lit if the segment joining it to the lamp does not contain any other tree; otherwise, it is shady. The structure of this lit/shady colouring involves both probability and arithmetic. Regarding probability, this is no surprise. As for arithmetic, it is rather expected to play a role as well because the definition of "lit" can naturally be phrased by using the word "multiple": a tree is lit if and only if the vector going from the lamp to this tree cannot be expressed as a multiple of a shorter vector with integer coordinates.

In this talk, we will be interested in the percolative properties of this random colouring. If we only keep the lit trees, how many infinite connected components are there? if we keep the shady trees instead? what happens if we work in other dimensions than 2?

[Joint work with Samuel Le Fourn and Mike Liu]

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