SYLVESTER PROBLEM: THE CASE WITH "A FLAT FLOOR"

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Abstract. Let us place n uniform points $U_1, \ldots, U_n$ at random in a compact convex set K of $\mathbb{R}^d$, and let $P_d(K, n)$ be the probability that these points form the vertex set of a convex polytope. The Sylvester problem is the name given to an optimization problem: Which compact convex sets (of non-empty interior) of $\mathbb{R}^d$ minimize $P_d(K, d+2)$? It is easy to see that $P_d(K, j) = 1$ for $j < d+2$, so that, $P_d(K, d+2)$ is the first non-trivial question. Maximization is also interesting, but considered solved (for $d+2$ points). It is conjectured that, in all dimension, the minimum is reached by simplexes... Except in 2D, where it is been a theorem since 1918 (Blaschke).

In this talk, we will be looking at a version of Sylvester's problem in which the question is modified: we will make the optimisation on the set of compact convexes having a common "flat floor" F; this means that K "lies above an hyperplane" and intersects it along a convex set F, the floor. The optimization problem is also modified: we require that the points $U_1, \ldots, U_n$ together with F are in a convex position, and we denote this value by $Q_F(K, n)$ (that is F and $U_1, \ldots, U_n$ are on the boundary of their convex hull). In this work, we solve (in any dimension), the first non-trivial question: given F, which compacts minimize and maximize $Q_F(K, 2)$? Time permitting, we will also give in 3D bounds for $Q_F(K, n)$.

[Joint work with Ludovic Morin, LaBRI, Bordeaux]

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