INTERSECTION PROBABILITIES FOR FLATS IN HYPERBOLIC SPACE

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ABSTRACT. Consider the d-dimensional hyperbolic space M_K^d of constant curvature $K < 0$ and fix a point o playing the role of an origin. Let $\mathbf{L}$ be a uniform random q-dimensional totally geodesic submanifold (called q-flat) in M_K^d passing through o and, independently of $\mathbf{L}$, let $\mathbf{E}$ be a random $(d-q+\gamma)$-flat in M_K^d which is uniformly distributed in the set of all $(d-q+\gamma)$-flats intersecting a hyperbolic ball of radius $u > 0$ around o. We are interested in the distribution of the random γ-flat arising as the intersection of $\mathbf{E}$ with $\mathbf{L}$. In contrast to the Euclidean case, the intersection $\mathbf{E} \cap \mathbf{L}$ can be empty with strictly positive probability. We determine this probability and the full distribution of $\mathbf{E} \cap \mathbf{L}$. Thereby, we elucidate crucial differences to the Euclidean case. Moreover, we study the limiting behaviour as $d \uparrow \infty$ and also $K \uparrow 0$. Thereby we obtain a phase transition with three different phases which we completely characterize, including a critical phase with distinctive behavior and a phase recovering the Euclidean results.

[Joint work with Panagiotis Spanos and Christoph Thäle.]

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