GrHyDy2024: Random spatial models

October 23-25, 2024

Location: Institut Mines Télécom Nord Europe,
Rue Guglielmo Marconi, 59650 Villeneuve-d'Ascq.

The workshop GrHyDy2024: Random spatial models aims at gathering during three days specialists of several topics in probability theory including random graphs, percolation, stochastic geometry and particle systems. Invited talks will cover an overview of the latest research advances on these subjects. The event is mainly funded by the ANR projet "Dynamical Hyperbolic Graphs" whose PI is Dieter Mitsche.

Wifi: SSID: wifiIMT
Pwd: wifiIMT!

Program

Wednesday October 23

10:00-10:40 : Welcome coffee

10:40-11:25 : Jean-François Marckert, Université de Bordeaux.

Sylvester problem: the case with "a flat floor"

Let us place n uniform points $U_1, \ldots, U_n$ at random in a compact convex set K of $\mathbb{R}^d$, and let $P_d(K, n)$ be the probability that these points form the vertex set of a convex polytope. The Sylvester problem is the name given to an optimization problem: Which compact convex sets (of non-empty interior) of $\mathbb{R}^d$ minimize $P_d(K, d + 2)$? It is easy to see that $P_d(K, j) = 1$ for $j < d+2$, so that, $P_d(K, d+2)$ is the first non-trivial question. Maximization is also interesting, but considered solved (for $d + 2$ points). It is conjectured that, in all dimension, the minimum is reached by simplexes... Except in 2D, where it is been a theorem since 1918 (Blaschke).

In this talk, we will be looking at a version of Sylvester's problem in which the question is modified: we will make the optimisation on the set of compact convexes having a common "flat floor" F; this means that K ?lies above an hyperplane? and intersects it along a convex set F, the floor. The optimization problem is also modified: we require that the points $U_1, \ldots, U_n$ together with F are in a convex position, and we denote this value by $Q_F(K, n)$ (that is F and $U_1, \ldots, U_n$ are on the boundary of their convex hull). In this work, we solve (in any dimension),

the first non-trivial question: given F, which compacts minimize and maximize $Q_F(K, 2)$? Time permitting, we will also give in 3D bounds for $Q_F(K, n)$.
[Joint work with Ludovic Morin, LaBRI, Bordeaux]

11:25-12:10 : Anna Gusakova, Universität Münster.

Random (Laguerre) tessellations

A (random) tessellation in $\mathbb{R}^d$ is a (random) countable locally-finite collection of convex polytopes, which cover the space and have disjoint interiors. One of the most well-studied and classical models is Poisson-Voronoi tessellation and in this talk we consider its generalized (weighted) version called Poisson-Laguerre tessellation. We will describe its construction and consider a number of properties including sectional properties and probabilistic description of the typical cell, as well as central limit theorems. We will also exploit the connection of the certain models with the extreme value theory.
[Joint work with Christoph Thäle, Zakhar Kabluchko and Mathias in Wolde-Lübke.]

12:10-14:00 : Lunch

14:00-14:45 : Arvind Singh, Université Paris Saclay.

Local limit of directed animals on the square lattice and intertwining

In this talk, I will discuss the local limit of rooted directed animals on the square lattice when their size grows to infinity. I will give two descriptions of the limiting object, one obtained by dropping dimer along a special random walk and the other as a branching-annihilating process that exhibits a remarkable intertwining property.
[Joint work with Olivier Hénard and Édouard Maurel-Segala]

14:45-15:30 : Irène Marcovici, Université Rouen Normandie.

A decentralised diagnosis method with probabilistic cellular automata

The decentralised diagnosis problem consists in the detection of a certain amount of defects in a distributed network. Here, we tackle this problem in the context of two-dimensional cellular automata with three states : neutral, alert and defect. When the density of defects is below a given threshold, we want the alert state to coexist with the neutral state while when this density is above the threshold, we want the alert state to invade the whole grid. We present two probabilistic rules to answer this problem. The first one is isotropic and is studied with numerical simulations. The second one is defined on Toom's neighbourhood and is examined with an analytical point of view. These solutions constitute a first step towards a broader study of the decentralised diagnosis problem on more general networks.
[Joint work with Nazim Fatès and Régine Marchand]

15:30-16:10 : Coffee breack

16:10-16:55 : Loïc Gassmann, Université de Fribourg.

Ordering and convergence of large degrees in Random Hyperbolic Graphs

A natural way to construct a random graph is by sampling points in a Euclidean space and connecting pairs of points that are separated by a distance smaller than a given threshold. In the Random Hyperbolic Graph, one rather use a hyperbolic space, leading to different behaviours in terms of percolation, clustering, degree distribution, etc. In particular, when the parameter α is larger than $1/2$, the graph exhibits key properties of complex networks, including a scale-free degree distribution.

In this talk, we focus on nodes with large degrees, which can be seen as the hubs of the graphs and play a major role in its connectivity properties. We will prove convergence in distribution of the degree sequence to a Poisson point process for all values of α. This result is obtained by proving an ordering property for the highest degrees: specifically, we show that (with high probability) the k nodes closest to the centre of the underlying space have the k highest degrees. In the scale-free regime, this can be made more precise by showing that (with high probability) the ranking of the nodes by increasing distance to the centre coincides with the ranking by decreasing degree up to rank $n^{1/(1+8\alpha)}$, but the two rankings differ beyond that point.

16:55-17:40 : Raphaël Butez, Université de Lille.

Transport and concentration for hyperuniform point processes in dimension 2

A (stationnary) point process is hyperuniform if the variance of the number of points inside a large window is neglictible with respect to the area of the window when its size goes to infinity. Perturbted lattices, eigenvalues of random matrices, determinantal point processes and zeros of gaussian analytic functions are examples of such processes. They all have in common the fact that the points are more evenly spread out than the ones of a Poisson point process. We will show that the empirical measures of a hyperuniform point process in a window converges towards the intensity measure and we will obtain concentration bounds in Wasserstein distances. The speeed of convergence is optimal and matches the best possible convergence rate in the convergence of atomic measures towards a measure with density.
[Joint work with Sandrine Dallaporta and David Garcia-Zelada, `https://arxiv.org/abs/2404.09549`]

17:40-18:10 : Carlos Martinez, IMCA.

The cover time of the giant component of the Random Geometric Graph with fixed radius

The cover time of a graph could be understood as the average time that it takes a random walker to visit all the vertices of the graph. In the Random Geometric Graph (RGG), the vertices appear from a Poisson Point Process of intensity 1 in the square $[-n/2, n/2]^d$, and the edges from connecting all the vertices that are at a distance at most r_n. It is known that there exists a connected giant component whp when $r_n > r_c$, where r_c is a critical constant. For $d > 1$, we prove that the cover time of the giant component of the RGG is of order $nlog^2(n)$ whp when $r_n = r$ is constant and $r > r_c$. The proof is based on the construction of logarithmic paths with "bad" connection and a grid-like structure of the giant component.

Thursday October 24

9:00-9:45 : Cristina Toninelli, Université Paris Dauphine.

Fredrickson-Andersen 2-spin facilitated model: sharp threshold

The Fredrickson-Andersen 2-spin facilitated model (FA-2f) on $\mathbb{Z}^d$ is a paradigmatic interacting particle system with kinetic constraints (KCM) featuring cooperative and glassy dynamics. For FA-2f vacancies facilitate motion: a particle can be created/killed on a site only if at least 2 of its nearest neighbors are empty. We will present sharp results for the first time, τ, at which the origin is emptied for the stationary process when the density of empty sites (q) is small. In any dimension $d \geq 2$ it holds

$$\tau \sim \exp\left(\frac{d\lambda(d,2)+o(1)}{q^{1/(d-1)}}\right)$$

w.h.p., with $\lambda(d, 2)$ the threshold constant for the 2-neighbour bootstrap percolation on $\mathbb{Z}^d$.

We will explain the dominant relaxation mechanism leading to this result, give a flavour of the proof techniques, and discuss further results that can be obtained via our technique for more general KCM, including full universality results in two dimensions.
[Joint work with I. Hartarsky and F. Martinelli]

9:45-10:30 : Baptiste Louf, Université de Bordeaux.

Counting with random walks

We are interested in an enumerative problem, namely counting geometric objects called combinatorial maps, which can be parametrized by two numbers: their size, and a topological parameter called the genus. We are interested in an asymptotic estimation of the number of these objects when both the size and the genus go to infinity. The original motivation is to use these counting estimates to study the geometry of random maps in large genus and their hyperbolic behaviour. While enumeration in one parameter is a very well studied topic with many powerful tools available, this problem is a case of bivariate enumeration, is a rather new topic with very few results known at the moment. Our method consists in studying a recurrence formula for these maps and modeling it by a random walk, forgetting completely about the combinatorics of the model.
[Joint work with Andrew Elvey-Price, Wenjie Fang and Michael Wallner]

10:30-11:10 : Coffee breack

11:10-11:55 : Eleanor Archer, Université Paris Dauphine.

Quenched critical percolation on Galton-Watson trees

We consider critical percolation on a supercritical Galton-Watson tree with mean offspring $m > 1$. It is well known that the critical percolation probability for this model is $1/m$ and that the root cluster has the distribution of a critical Galton-Watson tree. For this reason, many properties of the cluster are well understood, for example the probability of surviving for at least n generations, the limiting law of the size of the n-th generation conditioned on survival (the "Yaglom limit"), and convergence of the entire cluster to a branching process/stable tree. All these results are annealed, that is, we take the expectation with respect to the distribution of the tree and the percolation configuration simultaneously. The goal of this talk is to consider the quenched regime: are the same properties true for almost any realisation of the tree? We will see that this is indeed the case, although some scaling constants will depend on the tree.
[Joint work with Quirin Vogel and an ongoing project with Tanguy Lions]

11:55-12:40 : Arthur Blanc-Renaudie, Université de Rouen Normandie.

Scaling limit of bond percolation in high dimensions

In 2012, Addario–Berry, Broutin, and Goldschmidt proved that the largest connected components of critical Erdös–Rényi random graphs converge toward some continuum random graphs (sizes+geometry). Those results have then been extended for several other graphs, notably for the configuration model and multiplicative graphs. In this talk, we will see how we can extend those limits for bond percolation on the torus in high dimensions.
[Work in progress with Nicolas Broutin and Asaf Nachmias]

12:40-14:20 : Lunch

14:20-15:05 : Benedikt Jahnel, Weierstrass Institut Berlin.

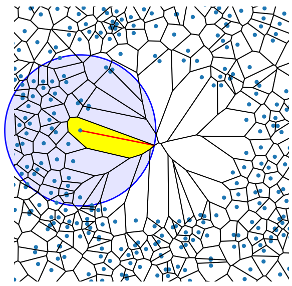
Cluster sizes in subcritical soft Boolean models

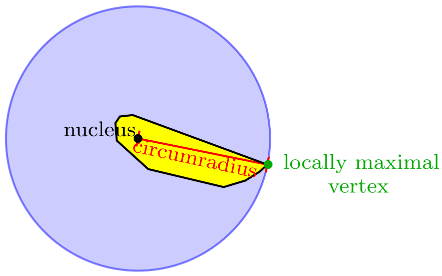
We consider the soft Boolean model, a model that interpolates between the Boolean model and long-range percolation, where vertices are given via a stationary Poisson point process. Each vertex carries an independent Pareto-distributed radius and each pair of vertices is assigned another independent Pareto weight with a potentially different tail exponent. Two vertices are now connected if they are within distance of the larger radius multiplied by the edge weight. We determine the tail behaviour of the Euclidean diameter and the number of points of a typical maximally connected component in a subcritical percolation phase. For this, we present a sharp criterion in terms of the tail exponents of the edge-weight and radius distributions that distinguish a regime where the tail behaviour is controlled only by the edge exponent from a regime in which both exponents are relevant.
[Joint work with Lukas Lüchtrath and Marcel Ortgiese.]

15:05-15:50 : Cécilia D'Errico, Université de Rouen Normandie et Paris Saclay.

Long and pointy Poisson-Voronoi cells: the distribution tail of the circumradius of the typical cell

Our model of interest is the *Poisson-Voronoi tessellation* generated by a homogeneous Poisson point process in $\mathbb{R}^d$. Our object of study is the *typical cell*, which corresponds to a cell chosen uniformly at random from the set of cells intersecting a large window, and more precisely its *circumradius*. The circumradius of a cell is the smallest radius that a ball centered on the nucleus must have to fully cover it, see figure.





We obtain an equivalent for the tail distribution of R_{circ}: when t tends to infinity,

$$\mathbb{P}(R_{\text{circ}} \geq t) \sim C_d t^{d(d-1)} e^{-\kappa_d t^d}.$$

The value of the constant C_d is provided by the new calculation of the expected volume of a certain random simplex in the unit ball.

15:50-16:30 : Coffee breack

16:30-17:15 : Sébastien Martineau, Sorbonne Université.

Arithmetic percolation

Plant a thin tree at each vertex of the square lattice. Then, pick a tree uniformly at random in a huge box, and replace it with a lamp. A tree is lit if the segment joining it to the lamp does not contain any other tree; otherwise, it is shady. The structure of this lit/shady colouring involves both probability and arithmetic. Regarding probability, this is no surprise. As for arithmetic, it is rather expected to play a role as well because the definition of "lit" can naturally be phrased by using the word "multiple": a tree is lit if and only if the vector going from the lamp to this tree cannot be expressed as a multiple of a shorter vector with integer coordinates.

In this talk, we will be interested in the percolative properties of this random colouring. If we only keep the lit trees, how many infinite connected components are there? if we keep the shady trees instead? what happens if we work in other dimensions than 2?
[Joint work with Samuel Le Fourn and Mike Liu]

17:15-18:00 : Benoît Henry, Institut Mines Télécom.

Percolation of the planar lattice two-neighbor graph

The two-neighbor graph is an oriented percolation model on a lattice in which each vertex independently picks two of its nearest neighbors at random and we open a directed edge towards those. We prove that on the two-dimensional square lattice, the origin is connected to infinity with positive probability. The proof rests on duality, exploration algorithm and enhancement arguments, as well as a comparison to iid bond percolation under constraints. As a byproduct, we show that an iid bond percolation constrained to avoid a given pattern has a strictly larger percolation threshold than $1/2$.
[Joint work with D. Coupier, B. Jahnel, J. Köppl]

19:30 : Social dinner at Brasserie de la Paix

Address: 25 Place Rihour, 59000 Lille.
https://www.brasserie-delapaix.fr/
(in the city center, very close to the metro station Rihour)

Friday October 25

9:00-9:45 : Maria Deijfen, Stockholm University.

Mean field stable matching

Consider a situation where a number of objects acting to maximize their own satisfaction are to be matched. Each object ranks the other objects and a matching is then said to be stable if there is no pair of objects that would prefer to be matched to each other rather than their current partners. We consider stable matching of the vertices in the complete graph based on i.i.d. exponential edge costs. Our results concern the total cost of the matching, the typical cost and rank of an edge in the matching, and the sensitivity of the matching and the matching cost to small perturbations of the underlying edge costs.

9:45-10:30 : Ercan Sönmez, Ruhr-Universität Bochum.

Intersection probabilities for flats in hyperbolic space

Consider the d-dimensional hyperbolic space M_K^d of constant curvature $K < 0$ and fix a point o playing the role of an origin. Let $\mathbf{L}$ be a uniform random q-dimensional totally geodesic submanifold (called q-flat) in M_K^d passing through o and, independently of $\mathbf{L}$, let $\mathbf{E}$ be a random $(d-q+\gamma)$-flat in M_K^d which is uniformly distributed in the set of all $(d-q+\gamma)$-flats intersecting a hyperbolic ball of radius $u > 0$ around o. We are interested in the distribution of the random γ-flat arising as the intersection of $\mathbf{E}$ with $\mathbf{L}$. In contrast to the Euclidean case, the intersection $\mathbf{E} \cap \mathbf{L}$ can be empty with strictly positive probability. We determine this probability and the full distribution of $\mathbf{E} \cap \mathbf{L}$. Thereby, we elucidate crucial differences to the Euclidean case. Moreover, we study the limiting behaviour as $d \uparrow \infty$ and also $K \uparrow 0$. Thereby we obtain a phase transition with three different phases which we completely characterize, including a critical phase with distinctive behavior and a phase recovering the Euclidean results.

[Joint work with Panagiotis Spanos and Christoph Thäle.]

10:30-11:10 : Coffee breack

11:10-11:55 : Elisabetta Candellero, Universita' degli Studi Roma Tre.

Properties of first passage percolation in hostile environment

Consider the following random competition model on a given graph G that is driven by two first-passage percolation processes FPP_1 and FPP_λ. Initially, FPP_1 occupies a single site and FPP_λ is dormant in seeds that are placed on the sites of G as a product of Bernoulli measures of parameter μ. Then, FPP_1 spreads through the edges of G at rate 1 and FPP_λ spreads from seeds at rate λ when that seed is attempted to be occupied by either FPP_1 or FPP_λ. Once a site is occupied by either process it remains occupied by that process forever. This model is known as first-passage percolation in a hostile environment (FPPHE) and was first introduced by Sidoravicius and Stauffer '19.

We establish that FPPHE is non-monotone in the sense that increasing μ or λ may increase the probability that FPP_1 occupies infinitely many sites by constructing a quasi-transitive graph where such behavior holds.

11:55-12:40 : Chinmoy Bhattacharjee, Universität Hamburg.

Sharp noise stability in continuum percolations via Spectra of Poisson functionals

In this talk, I will consider sharp noise stability of dynamical critical planer continuum percolation models, such as the Boolean model, which gives birth to a random geometric graph. Such a model has applications in the study of telecommunications networks and coverage optimization to name a few.

We consider the model under the Ornstein-Uhlenbeck (OU) dynamics. A critical planer percolation model is said to be noise stable under the OU dynamics if the *left-right occupied crossings* of large squares of side length L in the model are not sensitive to small noises in the underlying system. We show a sharp transition result : when the amount of noise tends to zero as $L \to \infty$ fast enough, then the model is stable under the noise, while if it doesn't tend to zero fast enough, the model is not stable. The main tool is a notion of spectral point process based on the chaos expansion of the crossing functionals, which parallels the corresponding notion of spectral samples in the discrete setting.
[Joint work with Giovanni Peccati and Yogeshwaran Dhandapani.]

12:40-14:20 : Lunch