Scaling limit of critical percolation on the high dimensional torus.

Arthur Blanc-Renaudie

with

Nicolas Broutin

Asaf Nachmias

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LMRS

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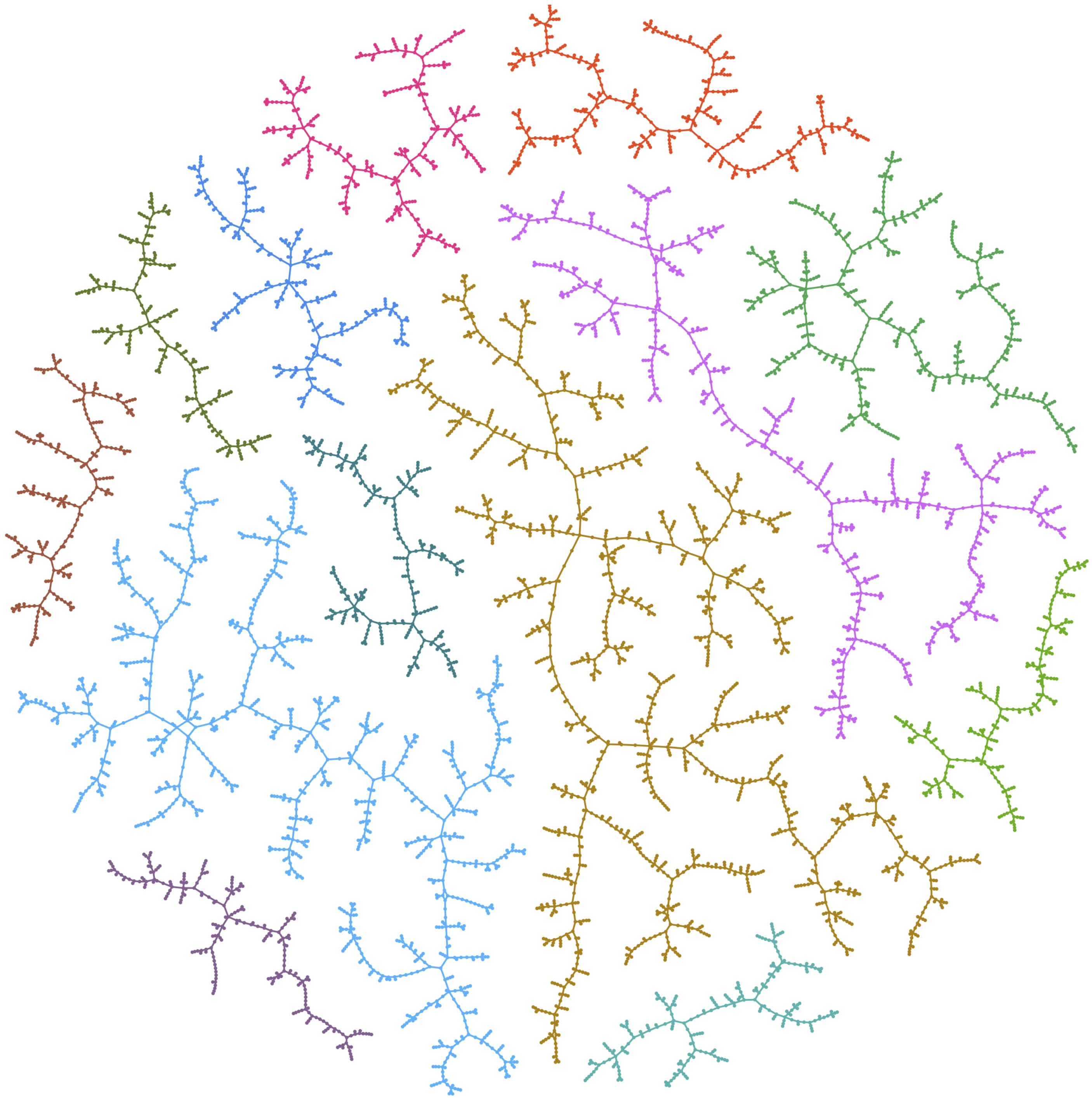
Introduction

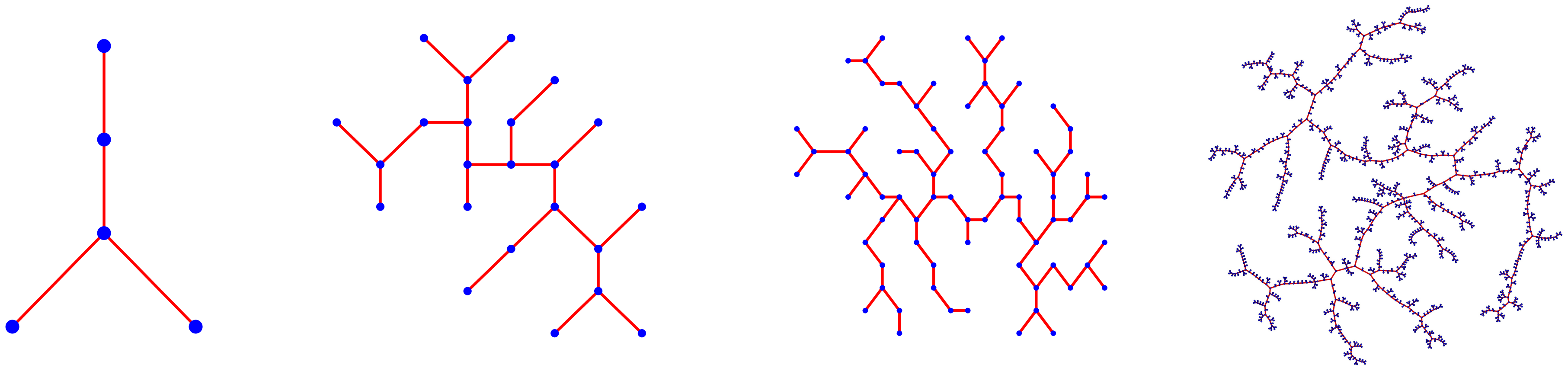


Scaling limits

Large random graphs

Distance : length shortest paths

$$\left(G_n, \frac{d_n}{\lambda_n}\right) \xrightarrow{\text{(law)}} (X, d).$$



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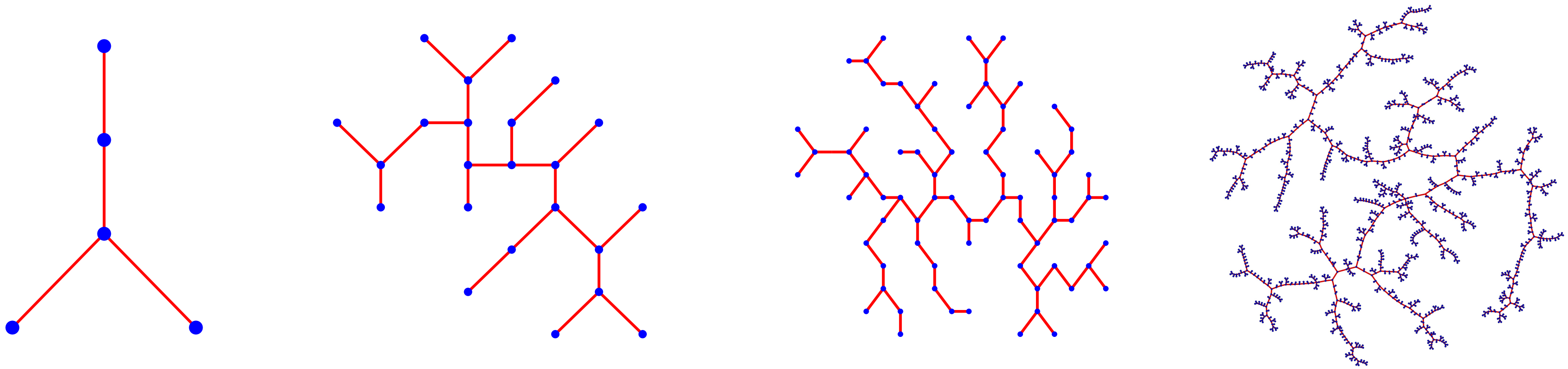

Formally : Topologies (GP/GHP...) $\hookrightarrow$ Distances between random vertices, diameter, height...

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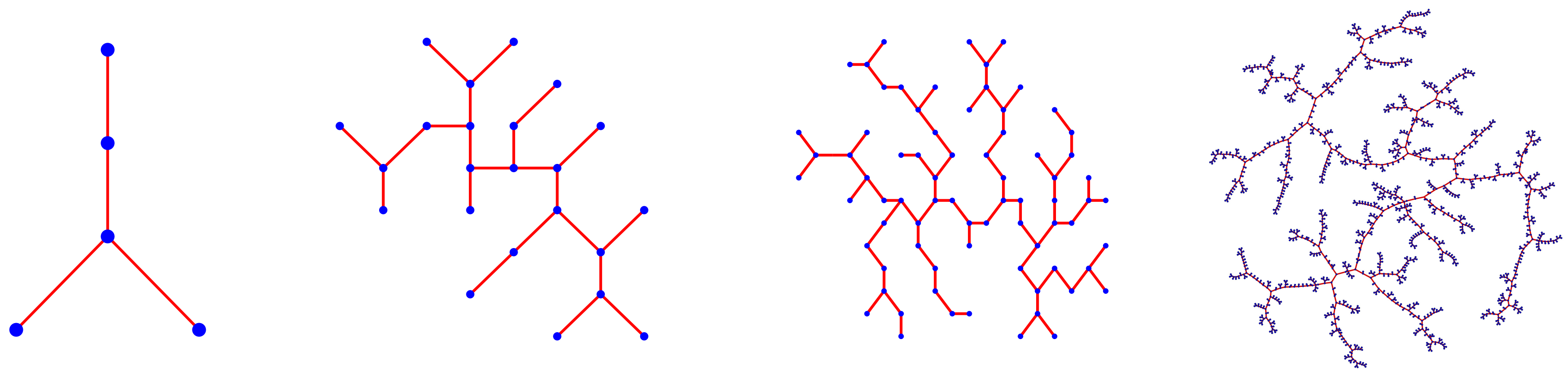

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Models :

- Combinatoric
- Informatic
- Statistical Physic

$\longrightarrow$ Universal Limits

Model

Torus: $(\mathbb{Z}/n\mathbb{Z})^d$ $n \to \infty$ High dim: $d > 6$ ($\gg$ or spreadout)

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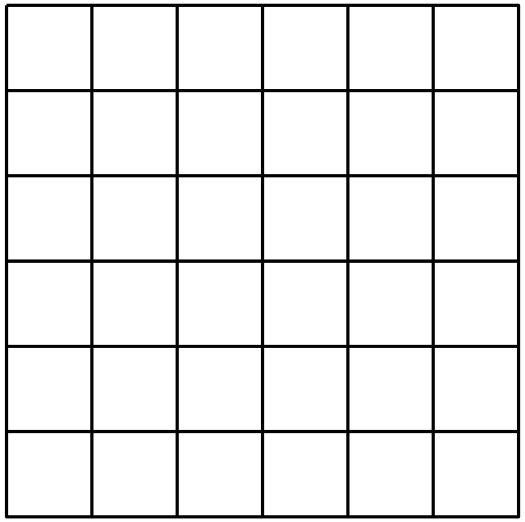

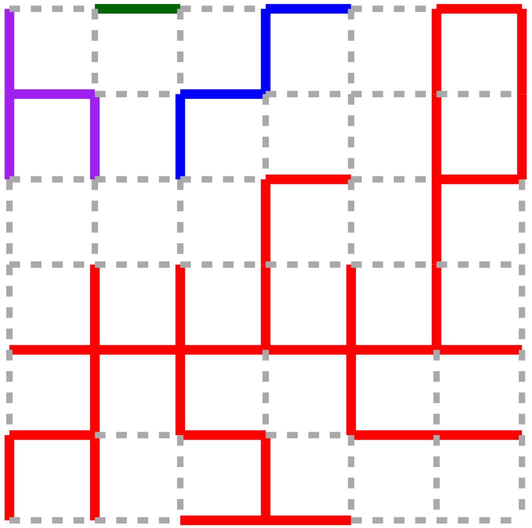

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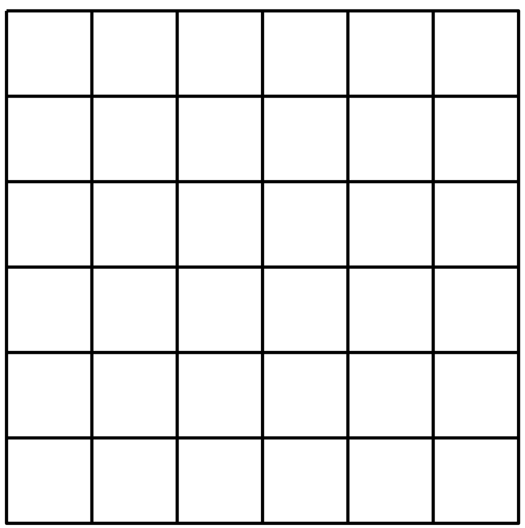
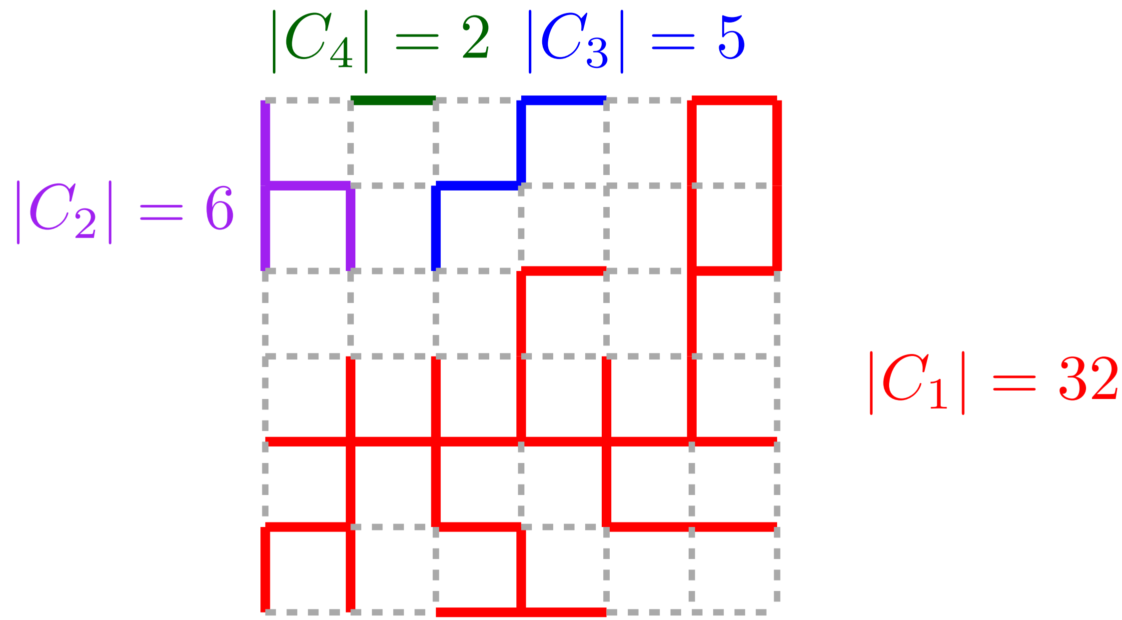




Clusters: connected components

Largest: $|C_1| > |C_2| > \ldots$ (#Vertices)

Conjecture

In high dimensions :

Percolation $\approx$ Erdös–Rényi

Yes

Very precisely around p_c

(Before : up to constants below p_c)

Result

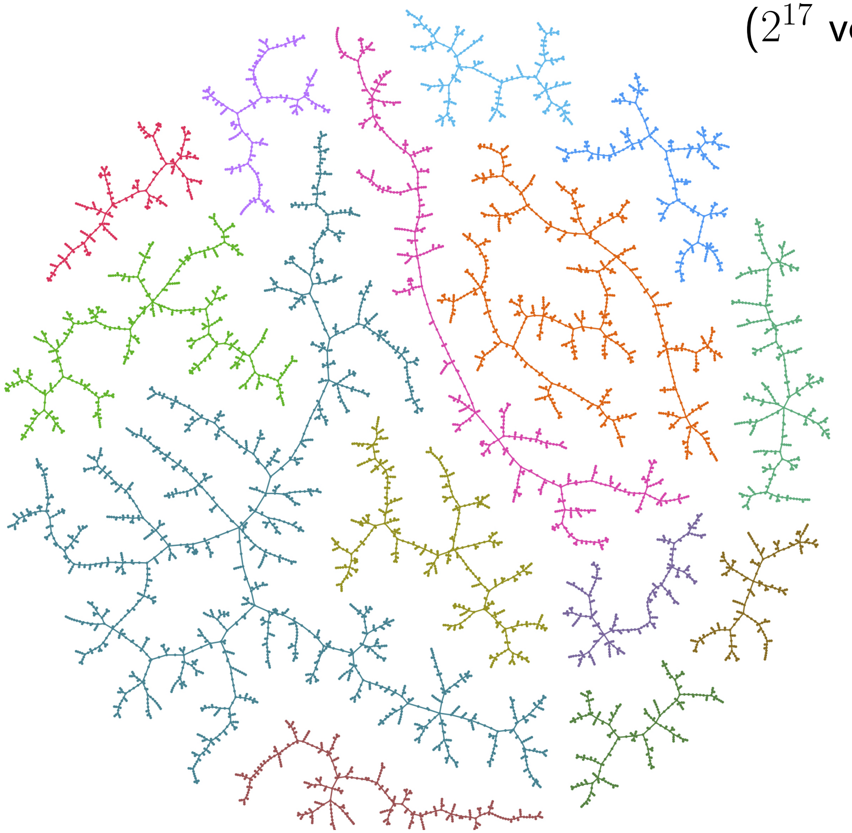
Theorem (Addario–Berry, Broutin, Goldshmidt, 12)

$G(n,p)$: $p_c(\lambda) = 1/n + \lambda n^{-4/3}$.

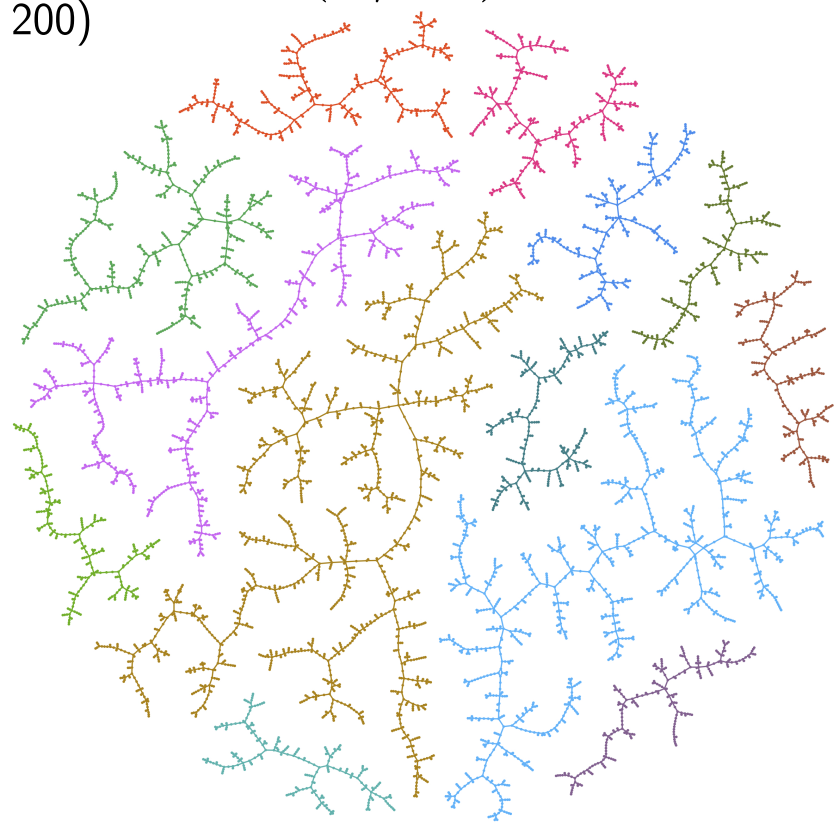
Scaling limit of $(C_i\ ,\ d_{\text{gr}}/n^{1/3}\ ,\ |C_i|/n^{2/3})_{i\in\mathbb{N}}$.

Critical Erdös–Rényi

$(2^{17}$ vertices ; $|C_i| \geq 200)$



$(\mathbb{Z}/2\mathbb{Z})^{17}$



Theorem (Blanc-Renaudie, Broutin, Nachmias)

Hypercube+Torus in high dim : Same scaling limit.

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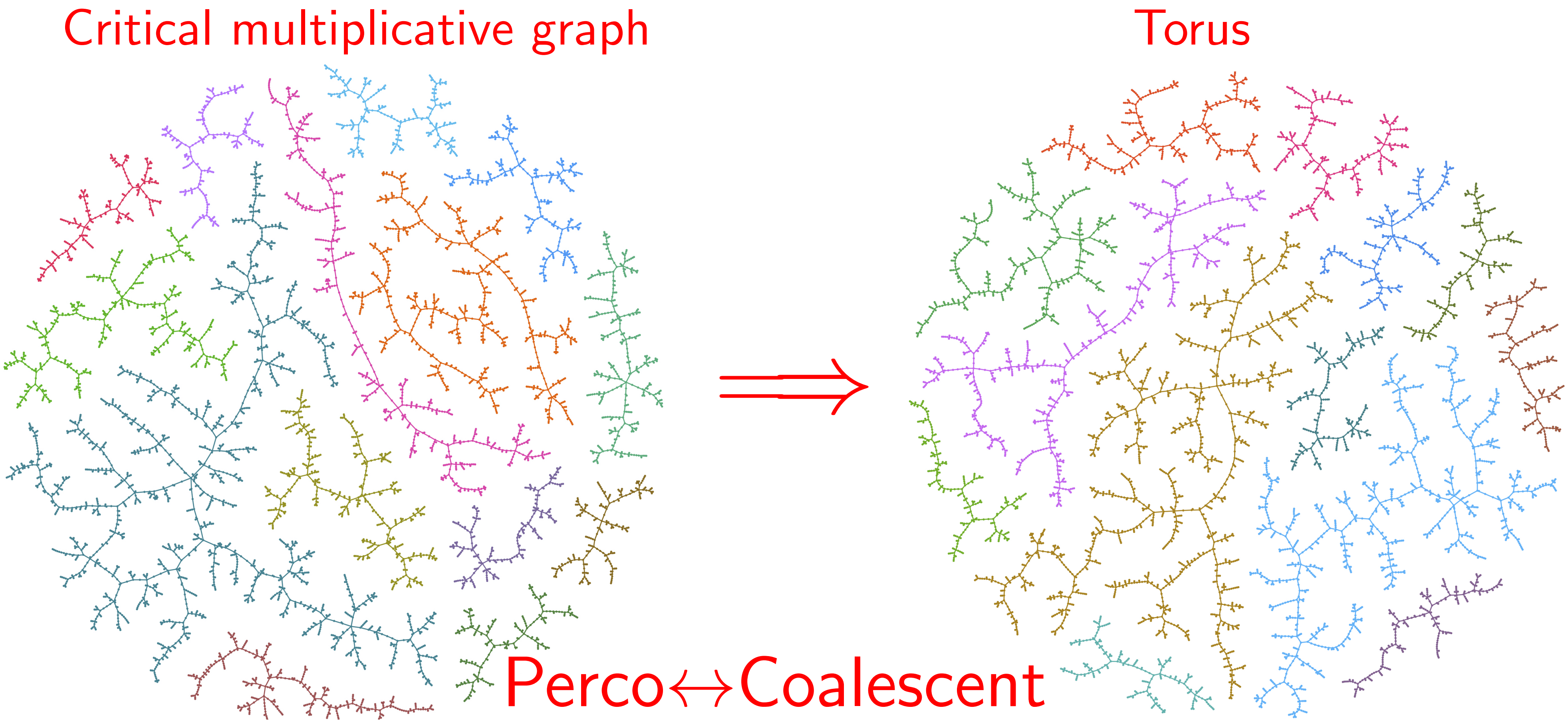

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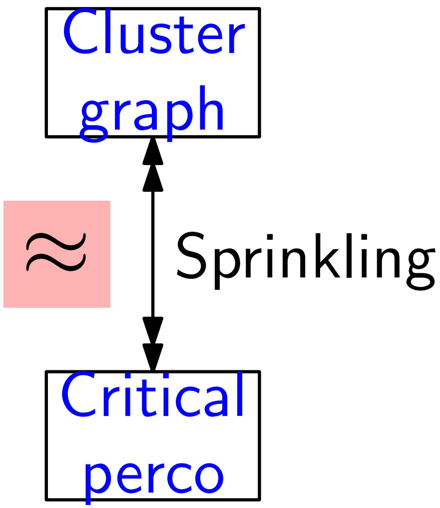
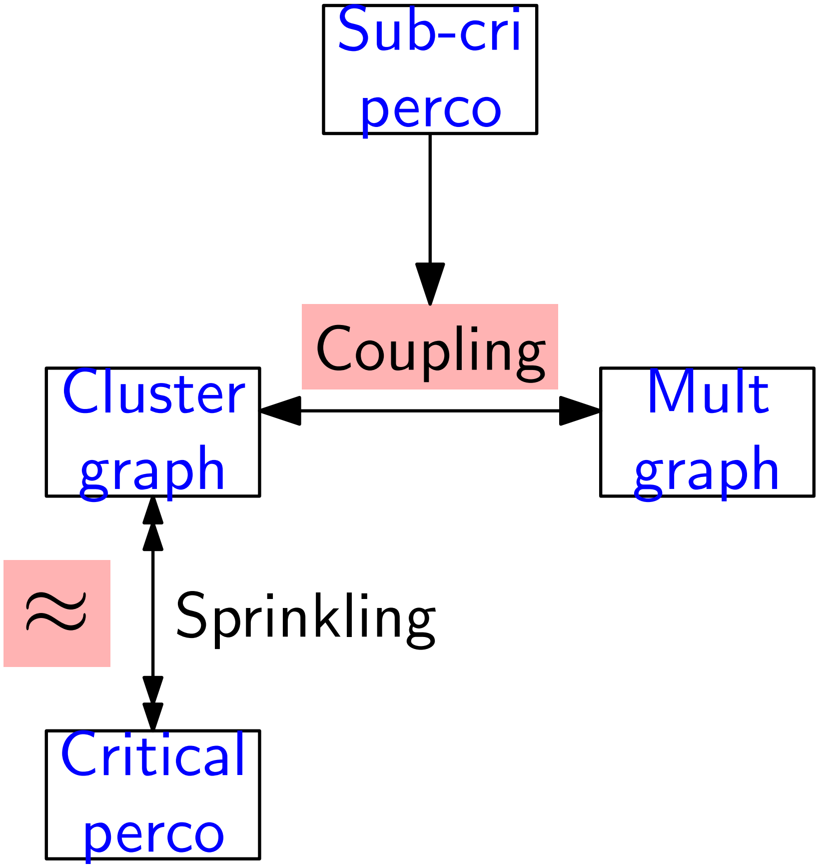
Plan

Hypercube



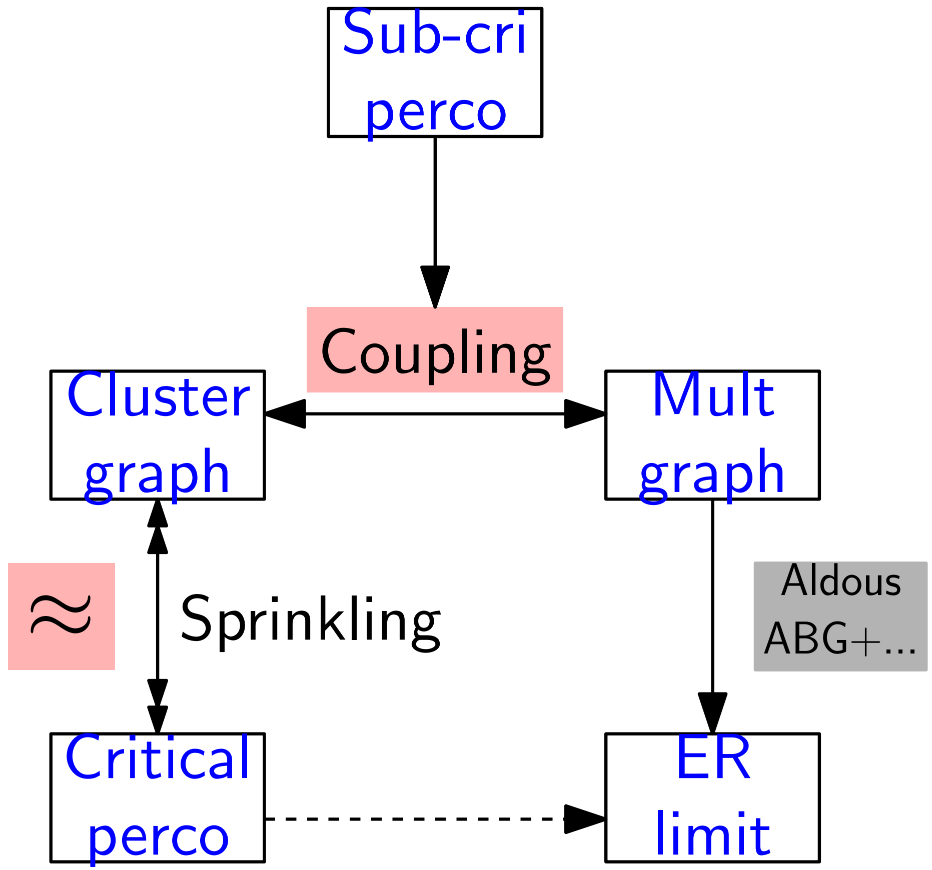
Plan

Hypercube



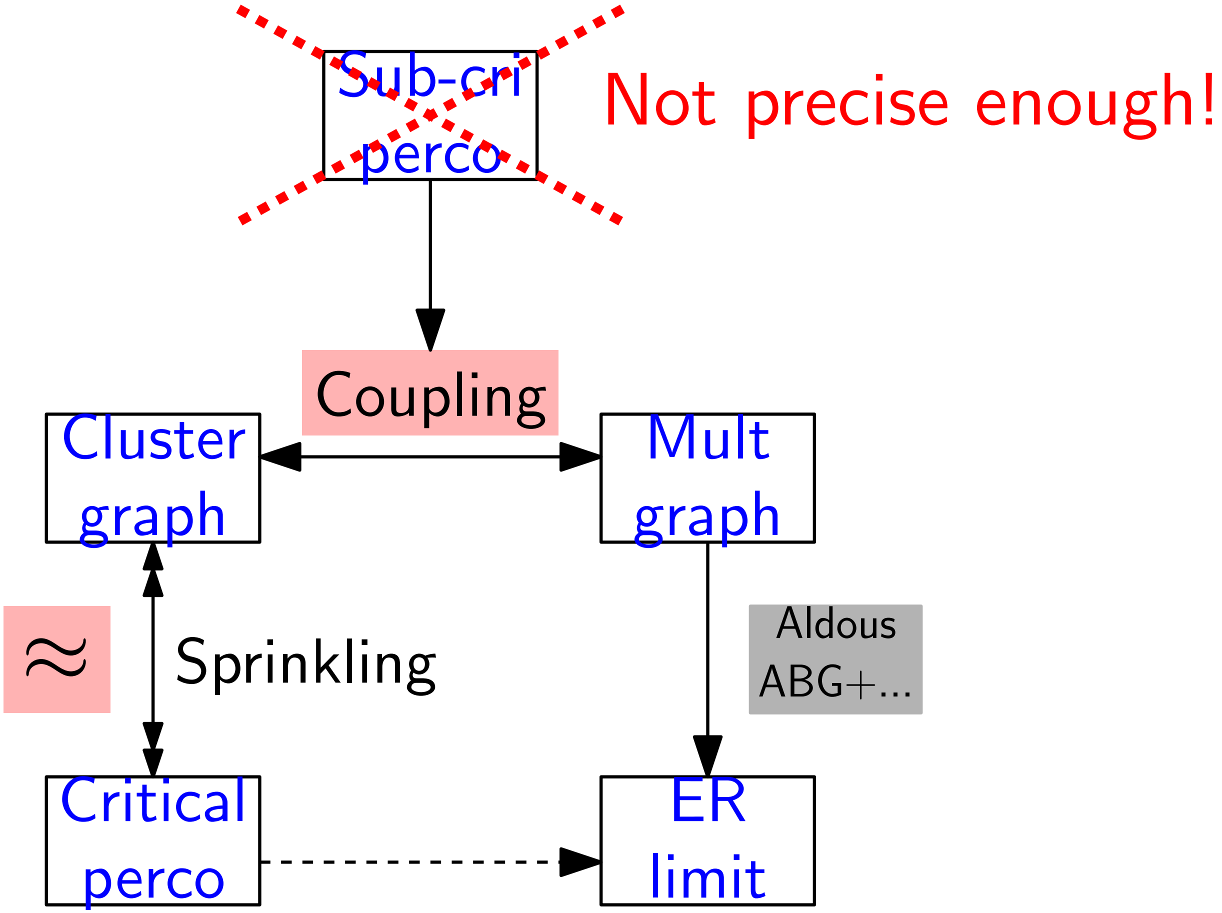
Plan

Hypercube



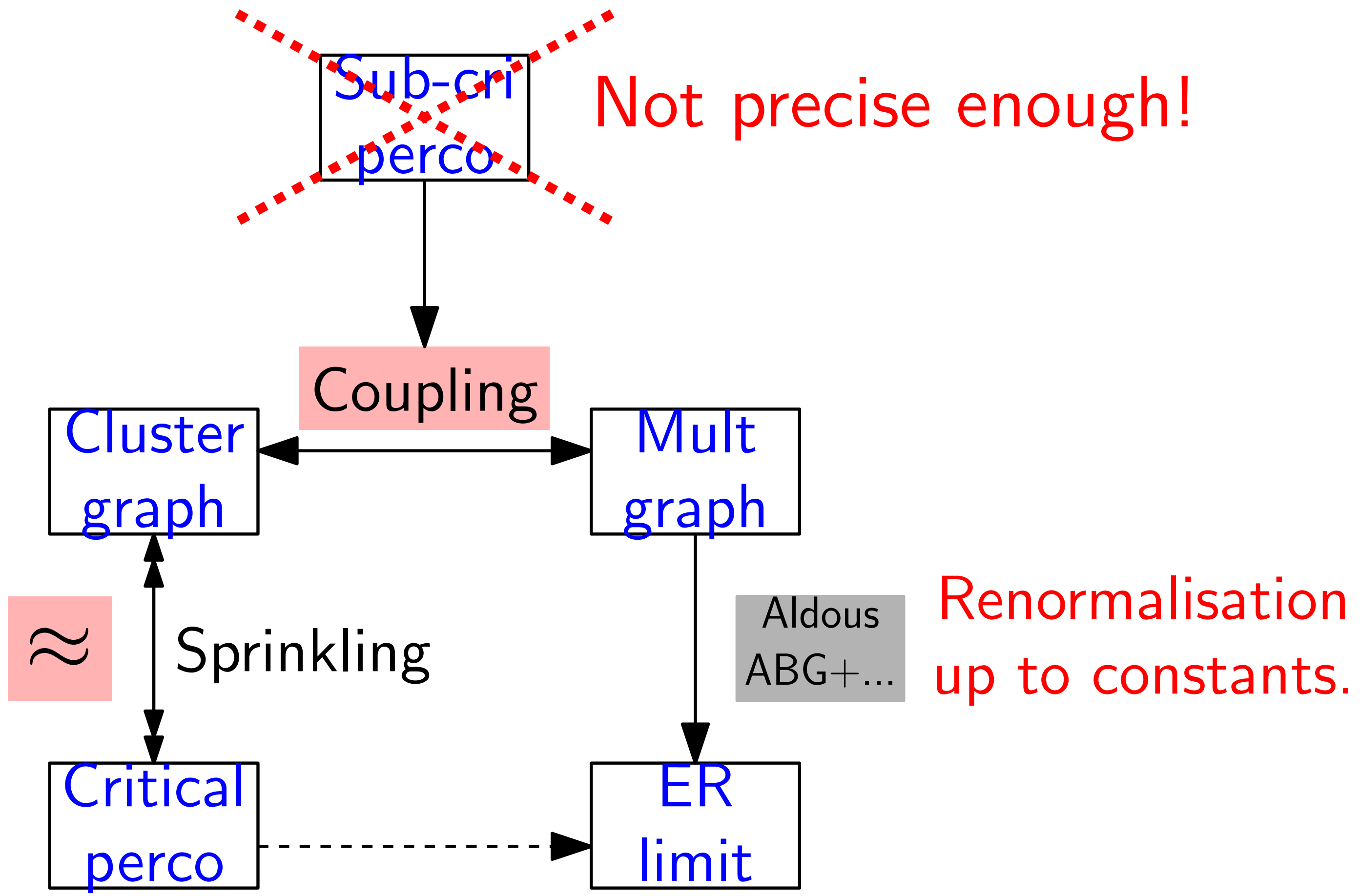
Plan

Torus


Sub-cri
perco
Not precise enough!
Coupling
Cluster
graph
Mult
graph
≈
Sprinkling
Aldous
ABG+...
Critical
perco
ER
limit

Plan

Torus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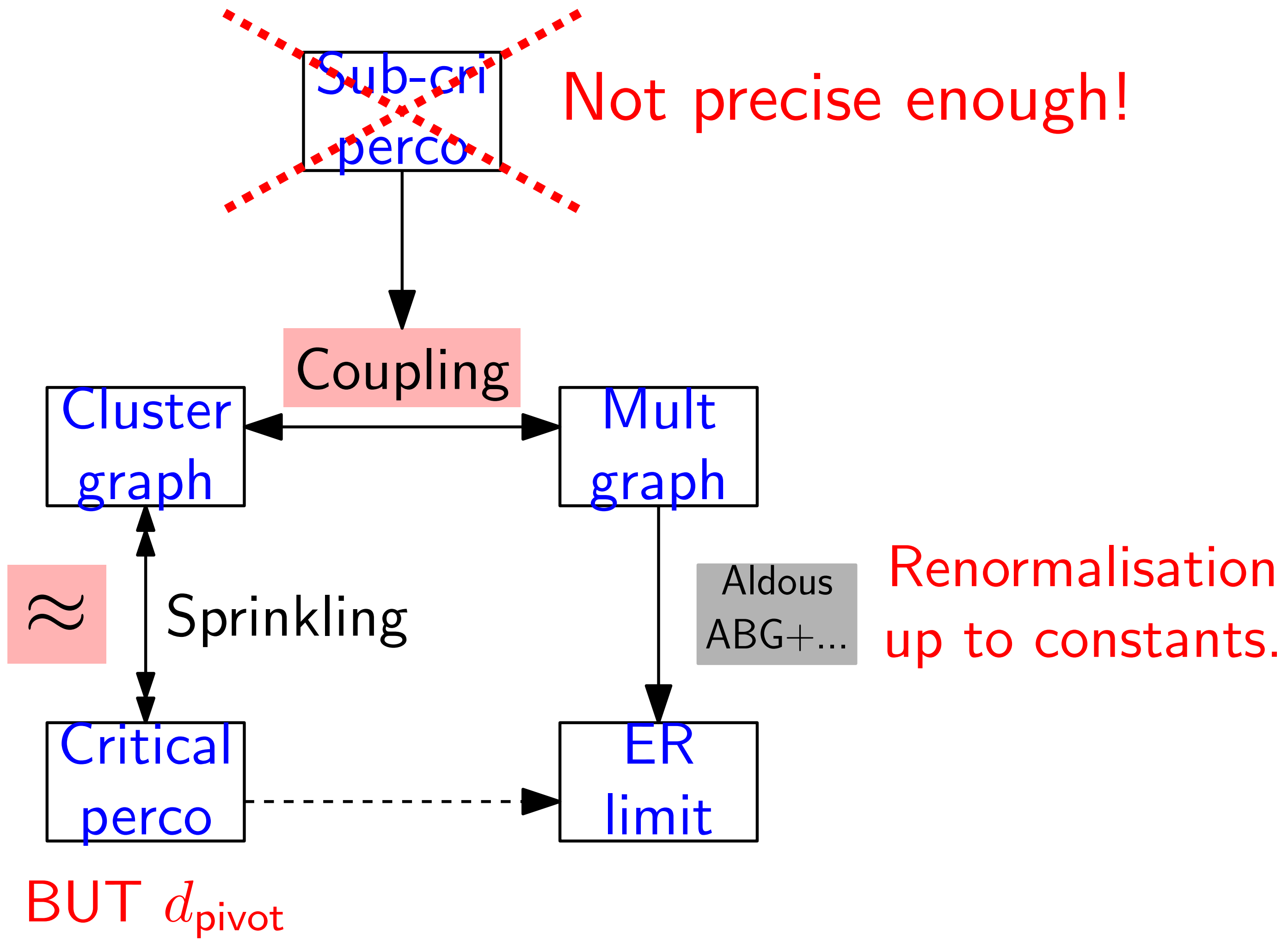

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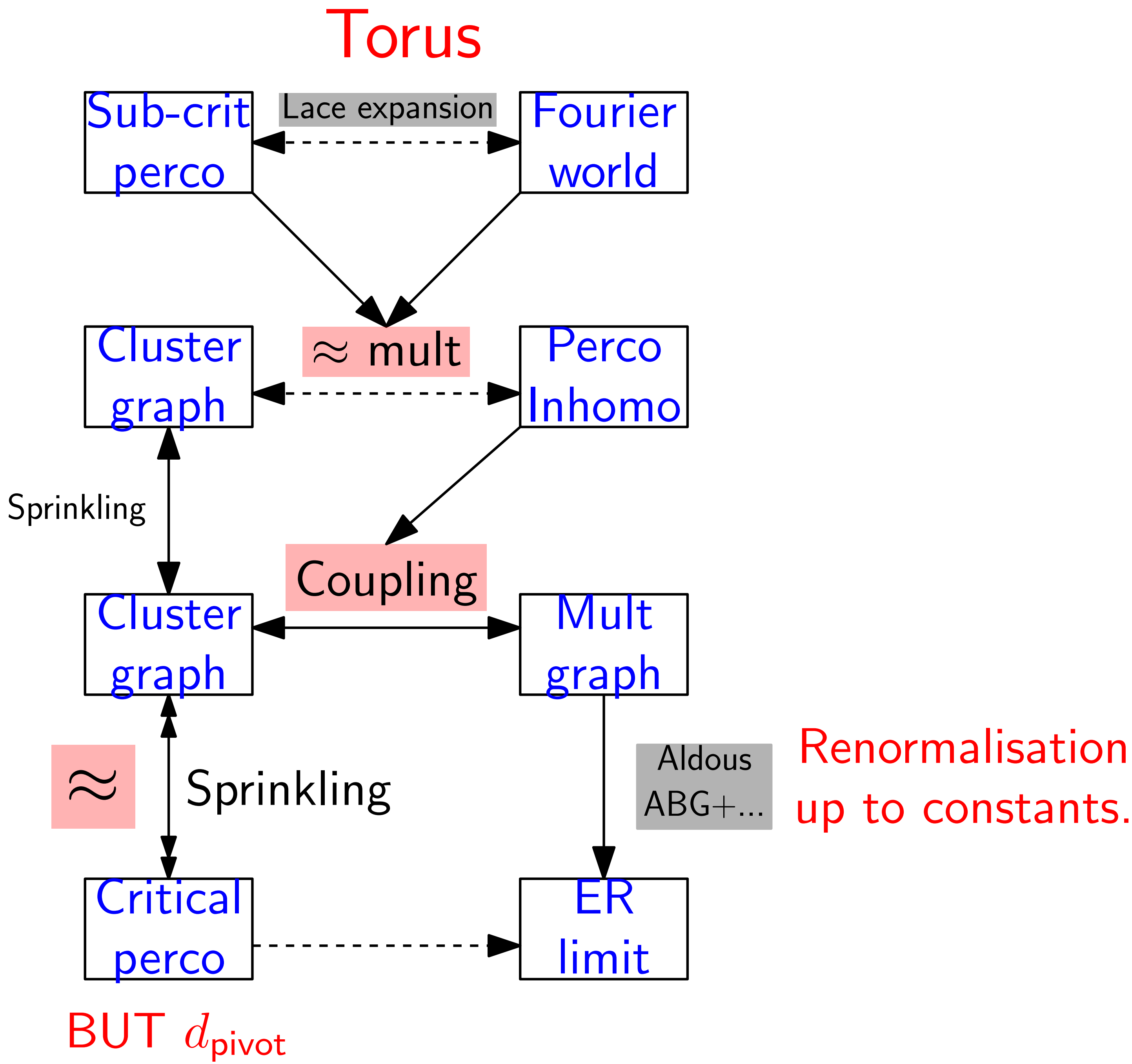
Torus
Sub-crit perco
Lace expansion
Fourier world
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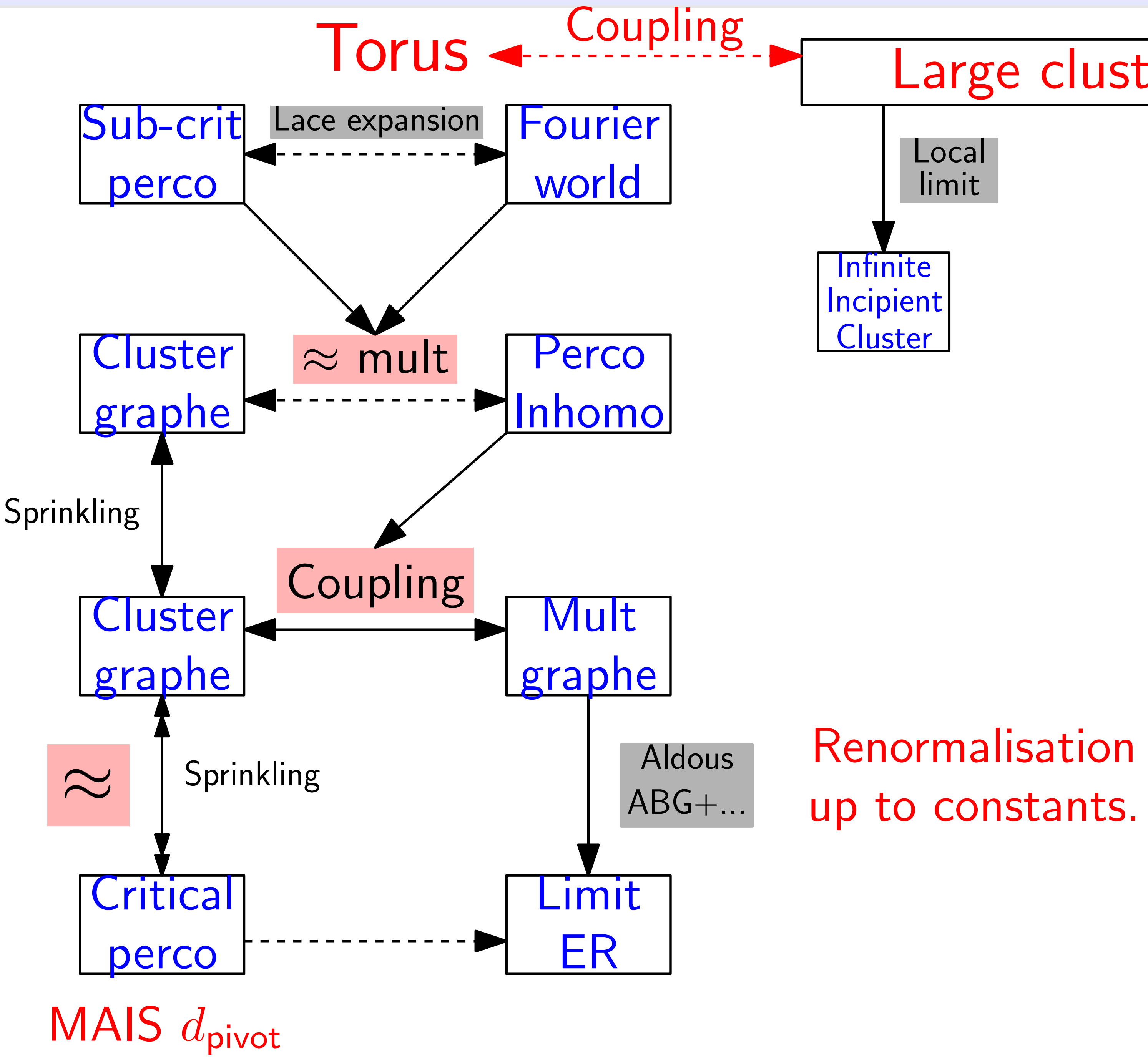

Torus
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Large clusters perco $\mathbb{Z}^d$
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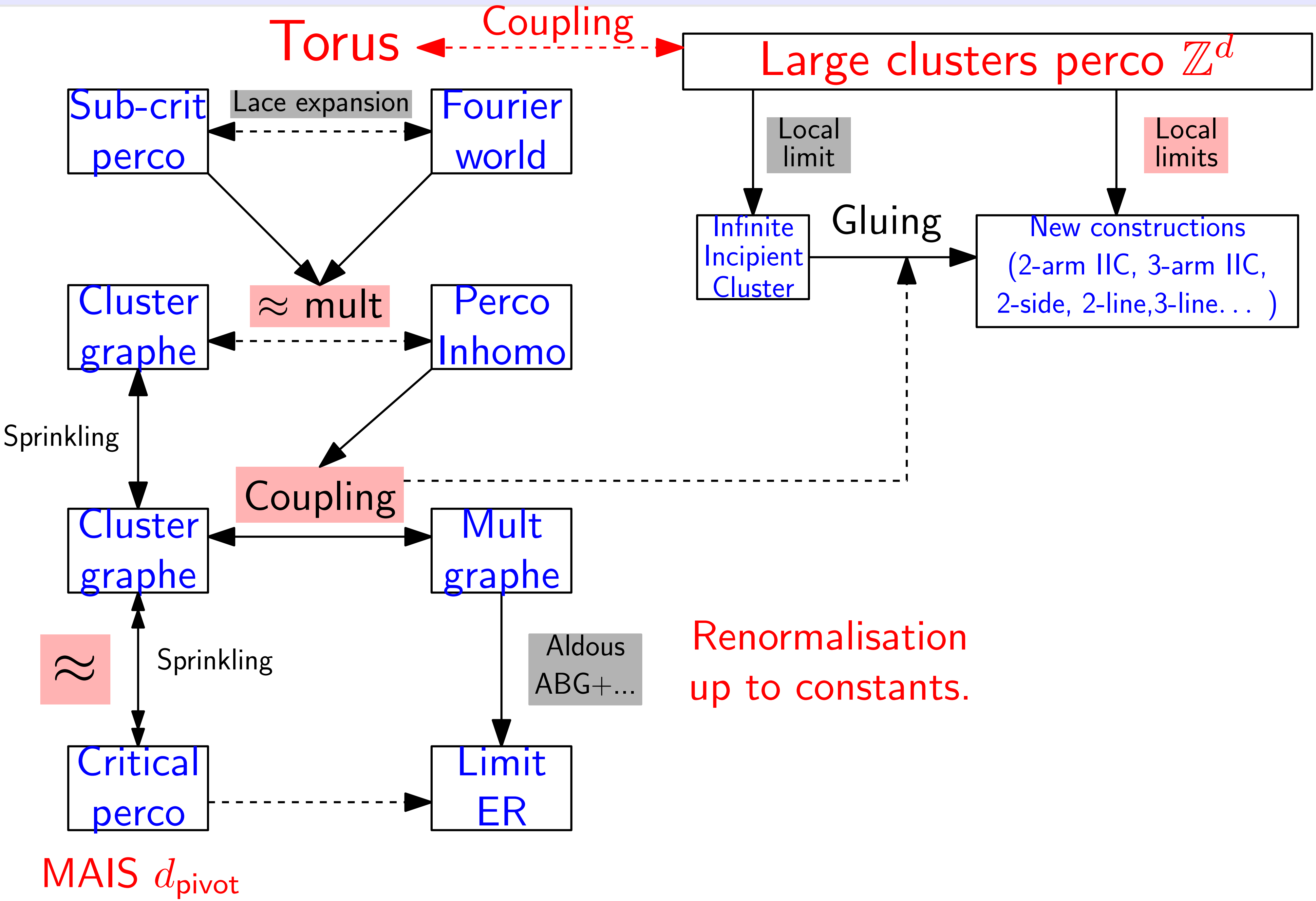
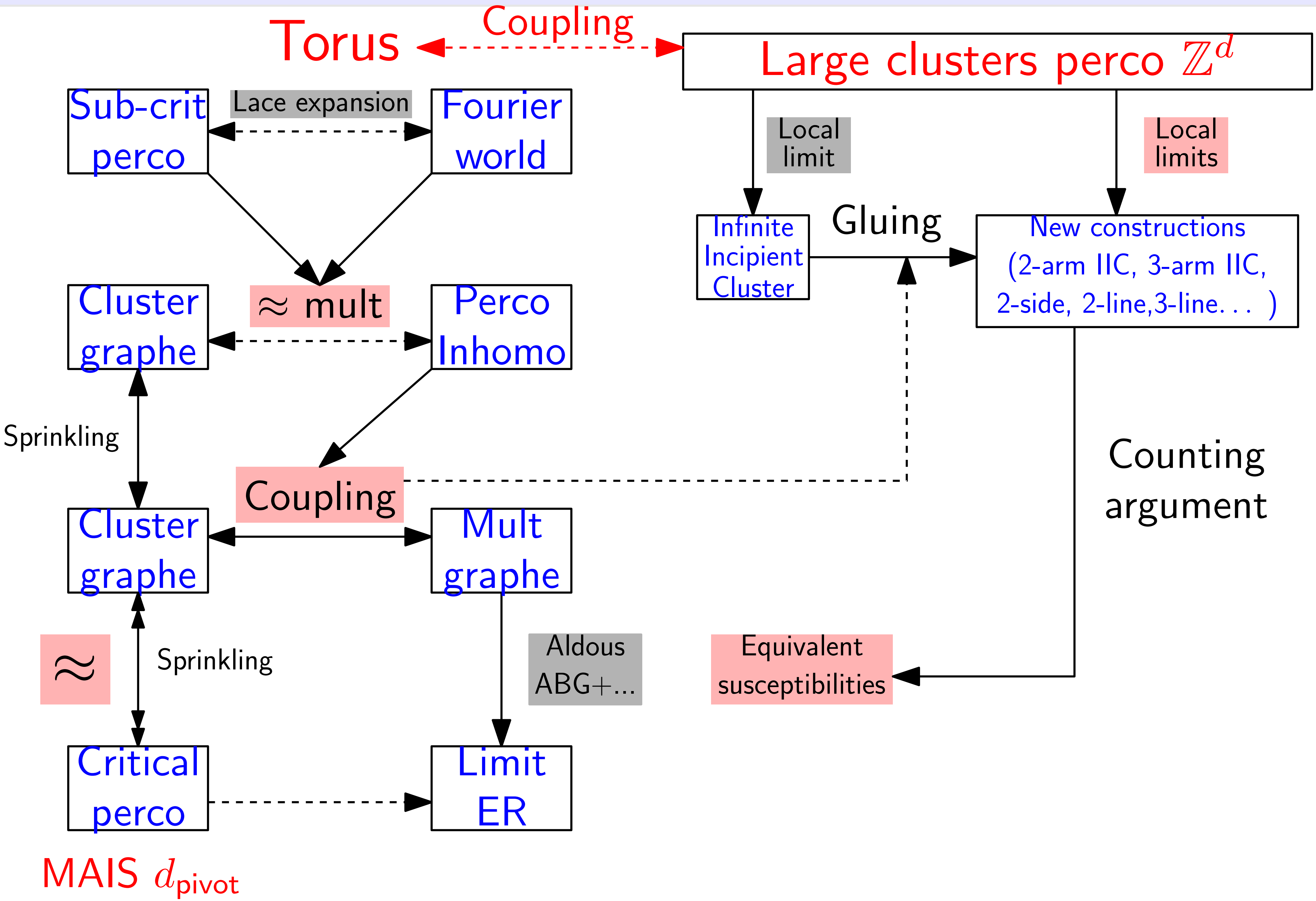


Renormalisation up to constants.

Plan



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New constructions (2-arm IIC, 3-arm IIC, 2-side, 2-line,3-line. . .)
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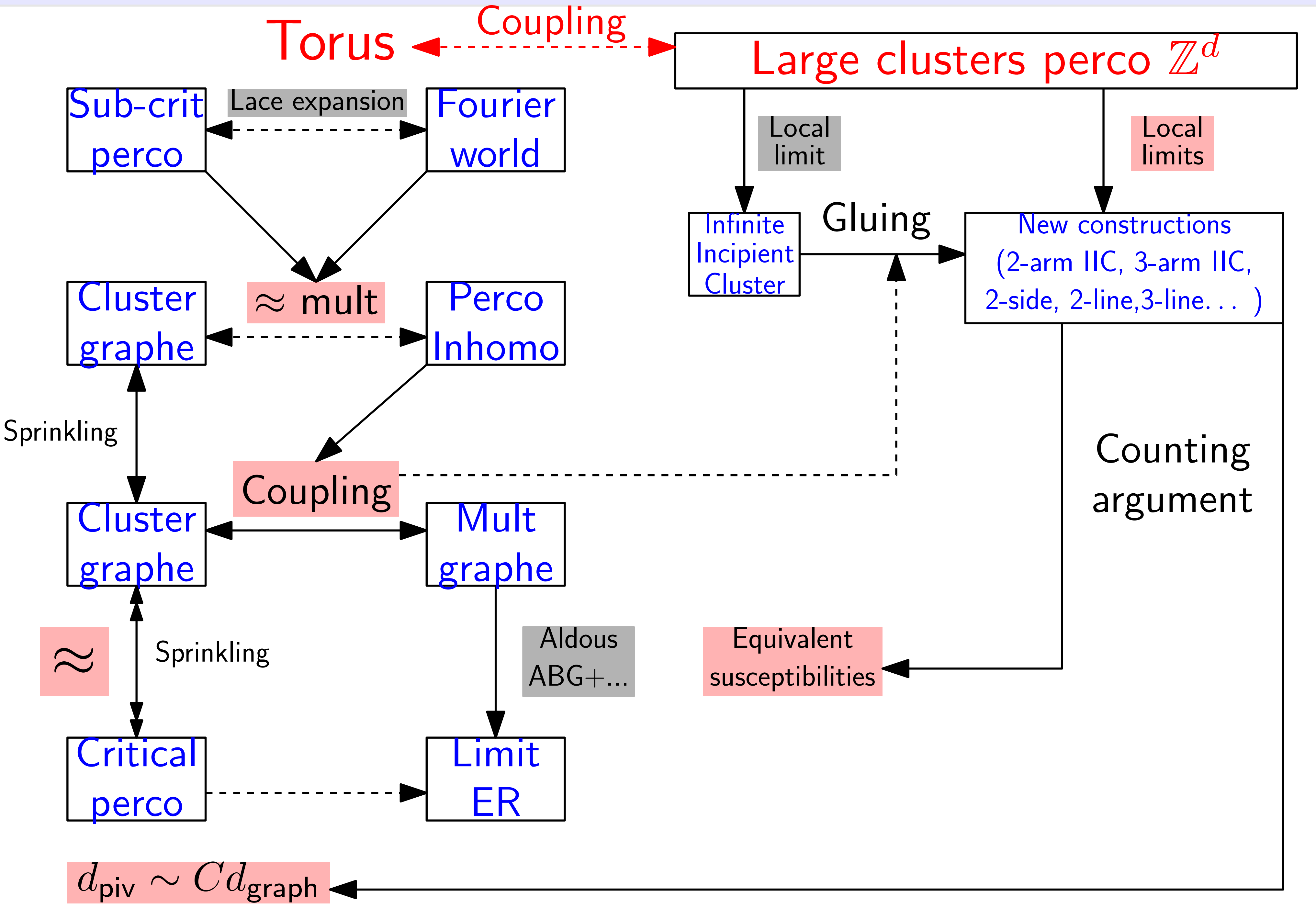
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$d_{\text{piv}} \sim C d_{\text{graph}}$

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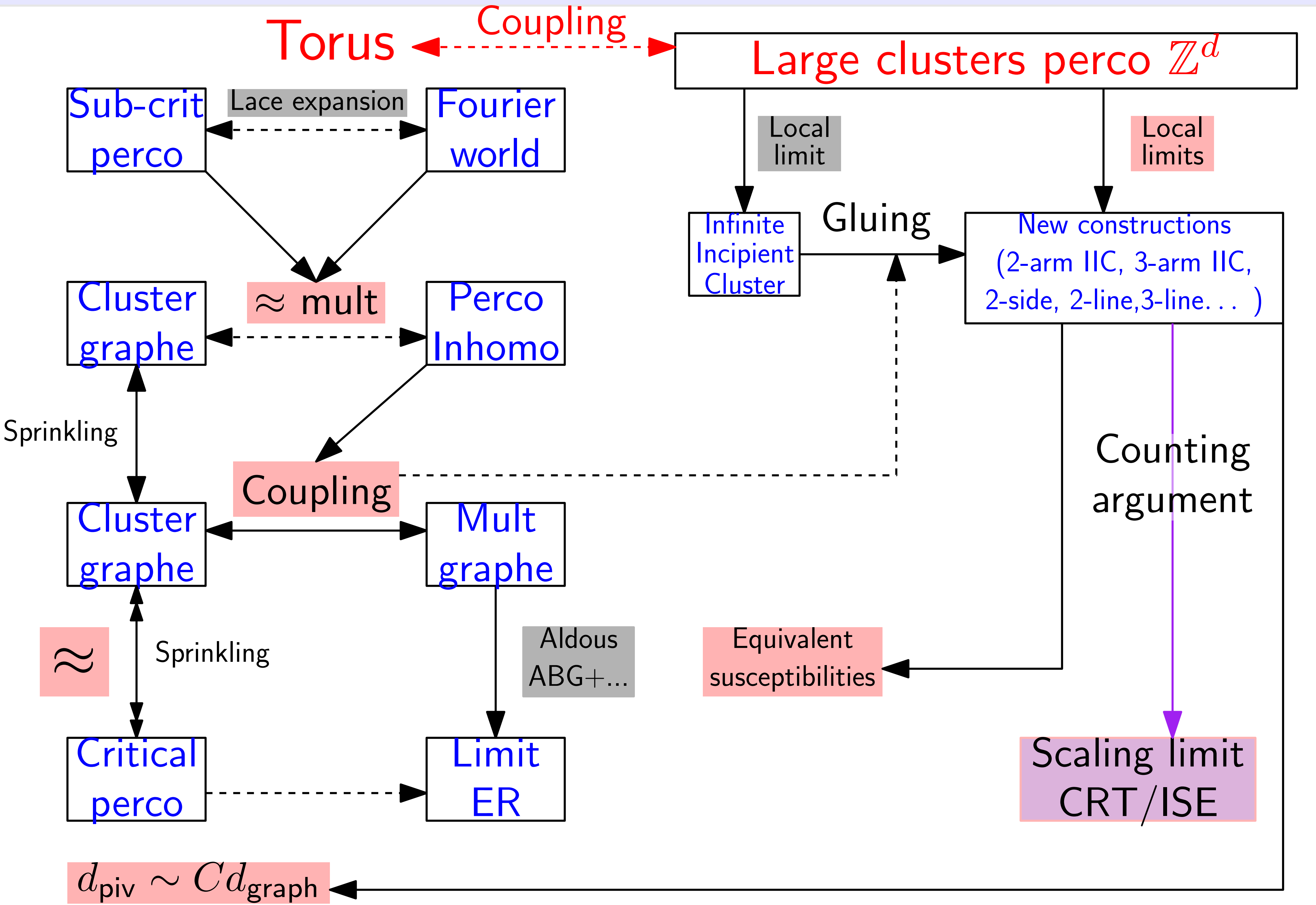
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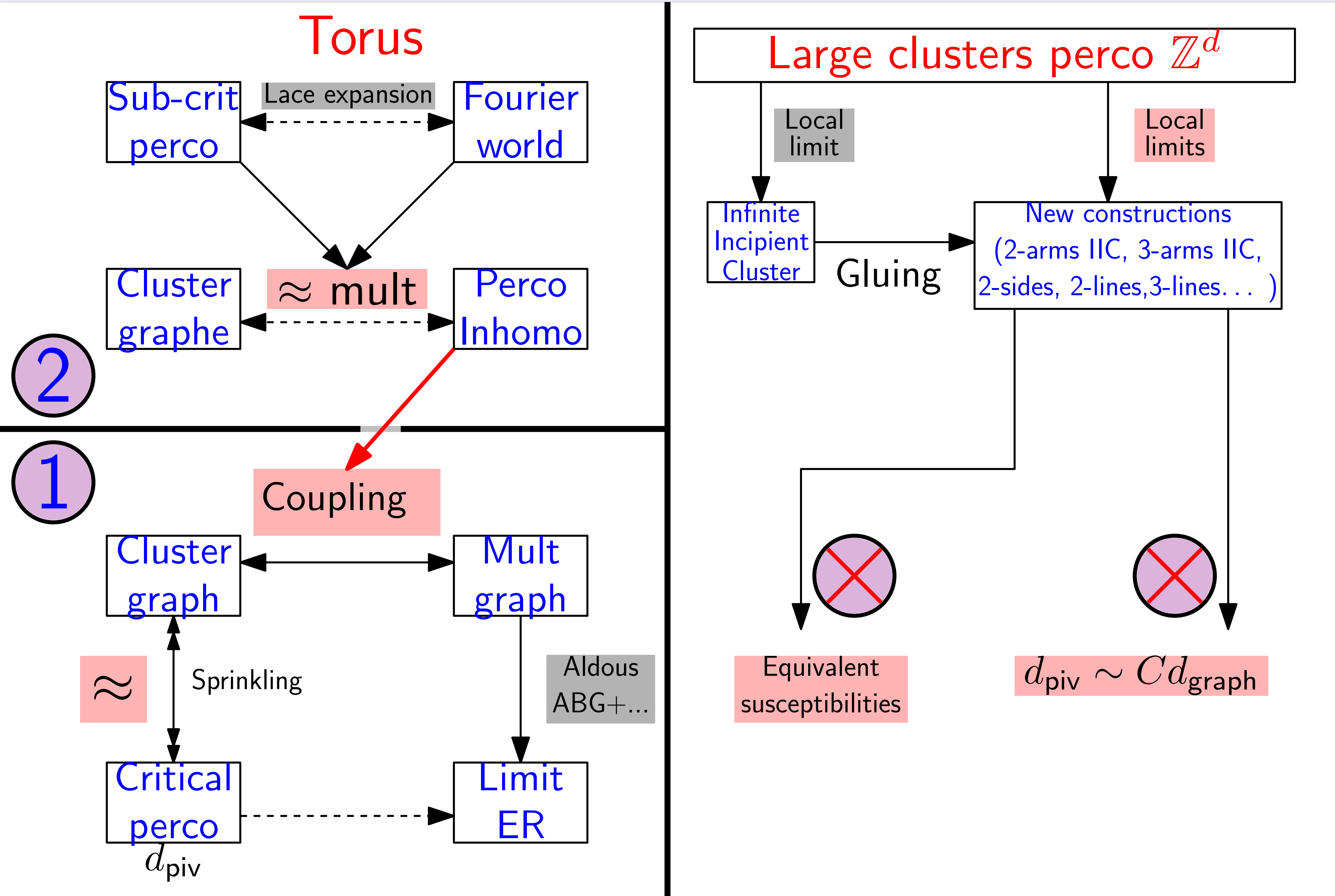
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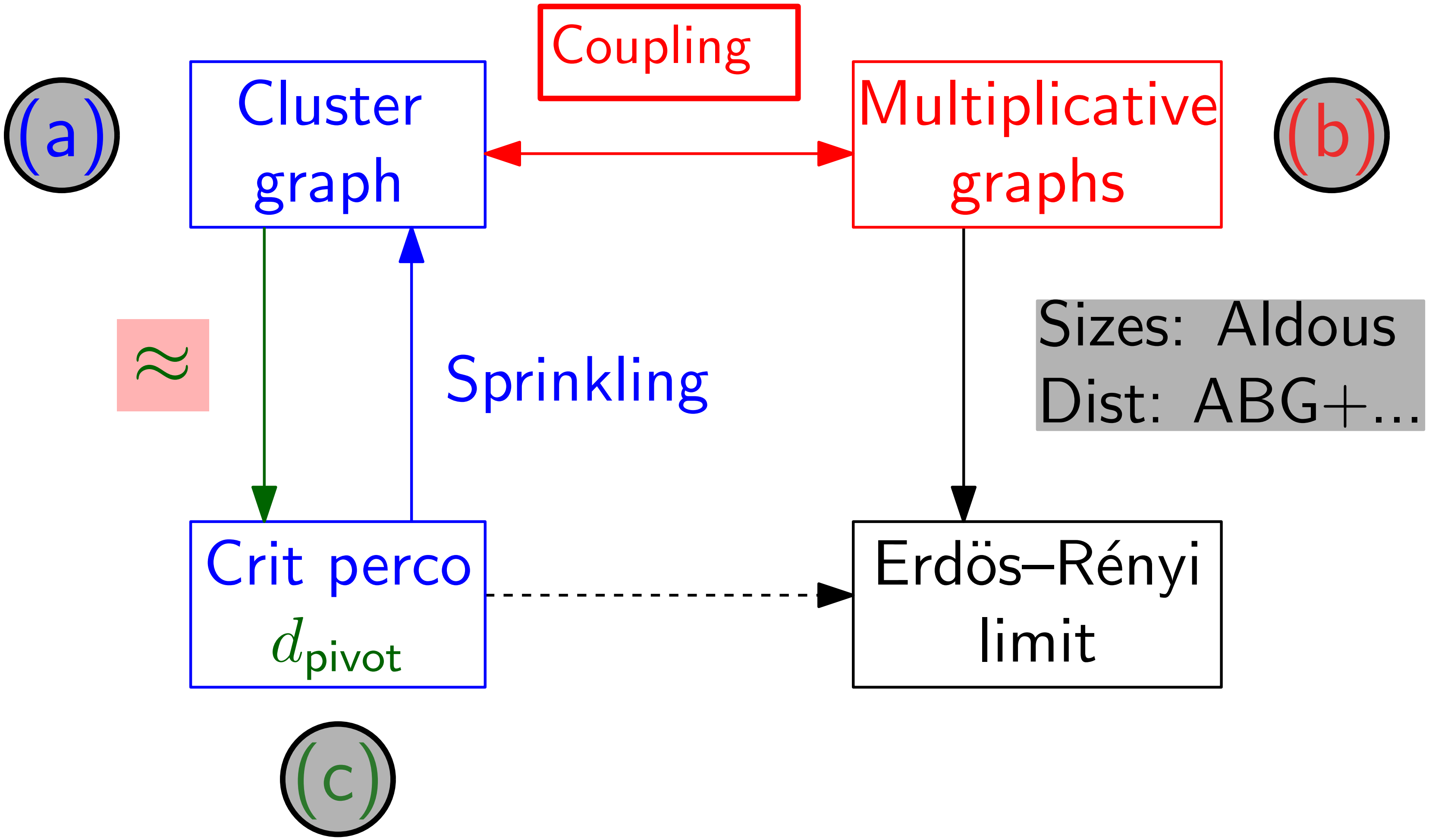
I Link percolation/×-graphs

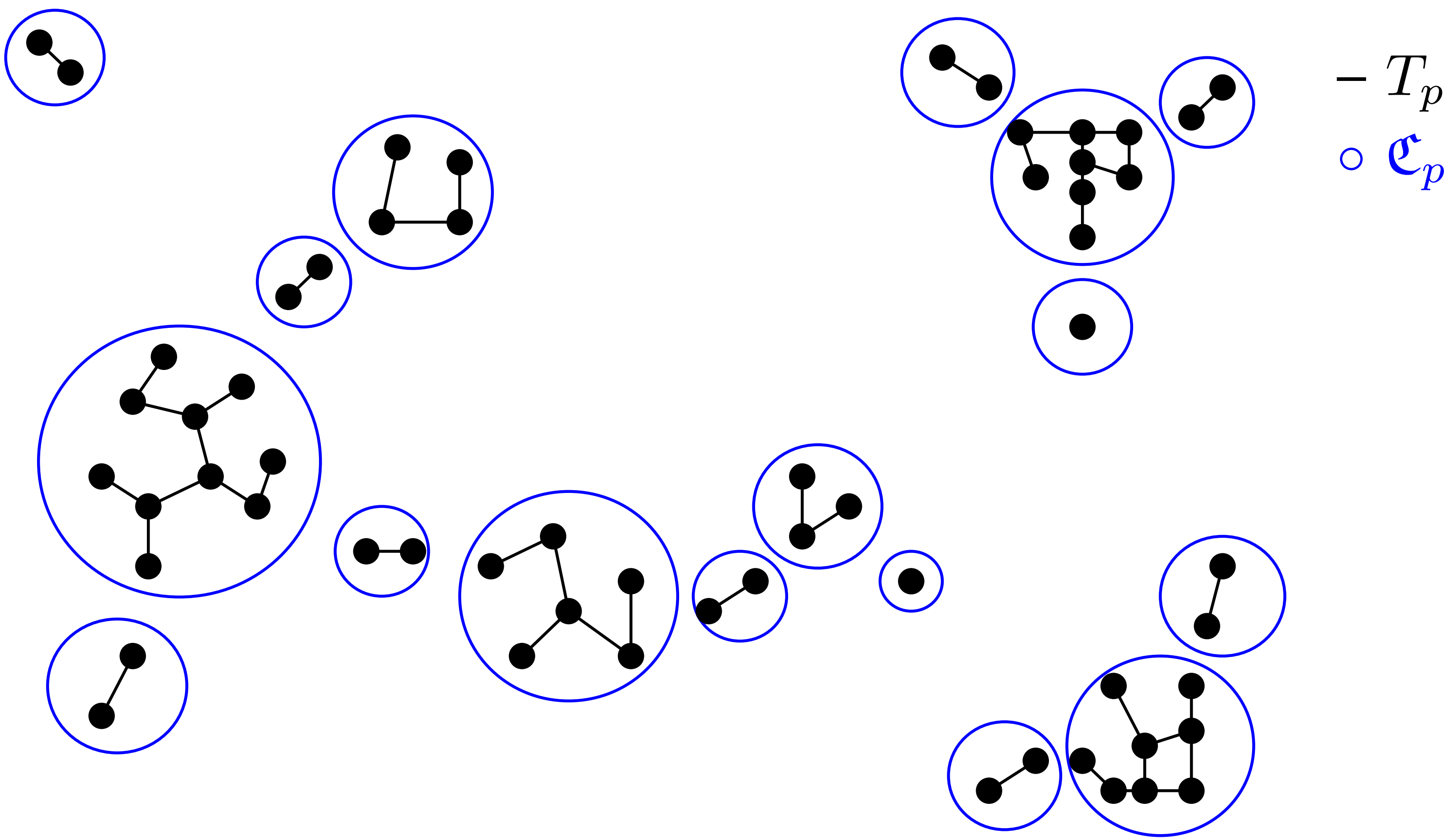


Cluster graph

Sprinkling: $p < p'$. Study $T_{p'}$ conditionally T_p

Definition (Cluster graph $T_{p,p'}$)

- Vertices: $\mathfrak{C}_p$ the clusters of T_p.
- $(A, B) \in T_{p,p'}$ iff there is an edge of $T_{p'}$ between A, B.



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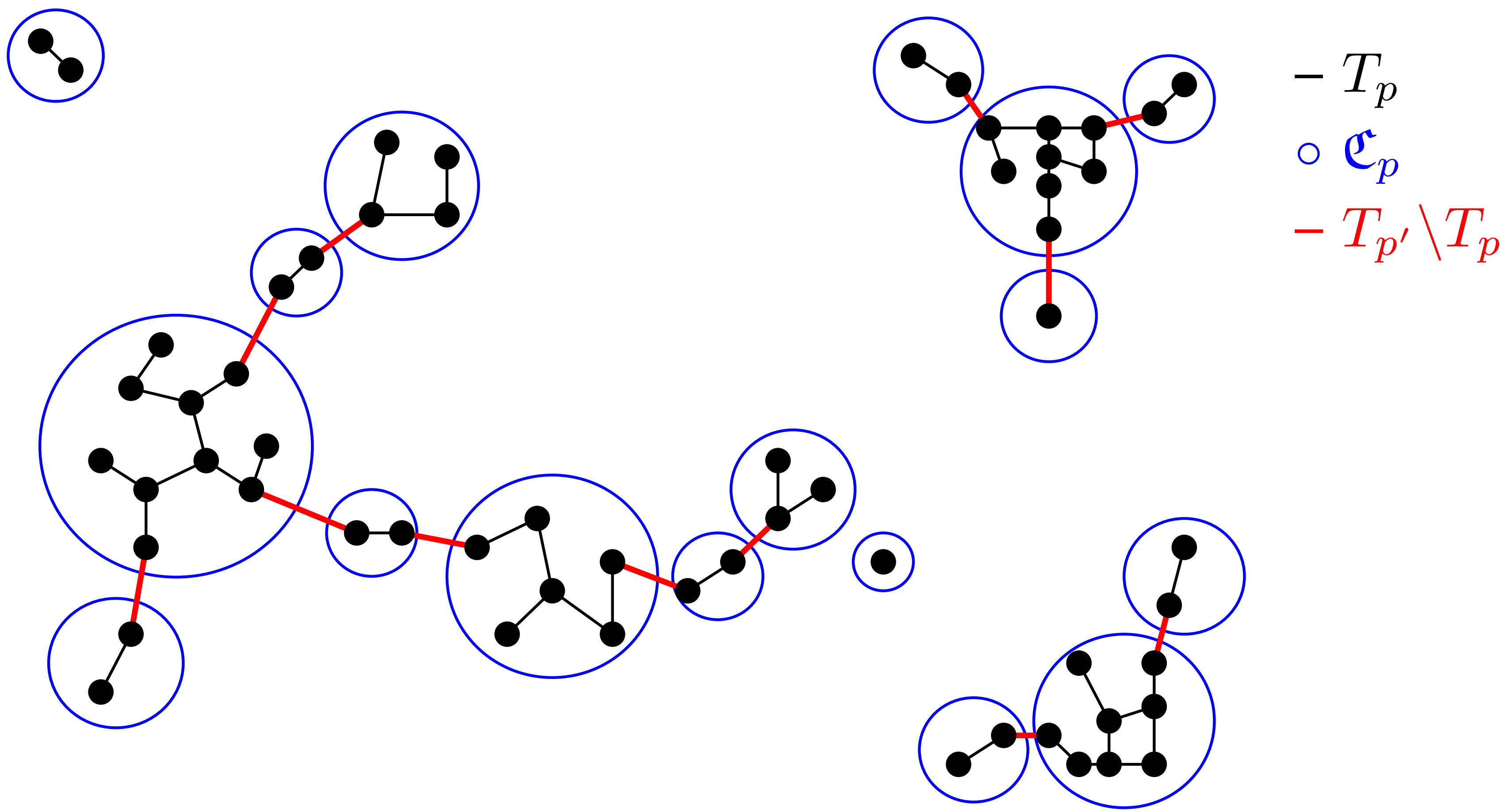

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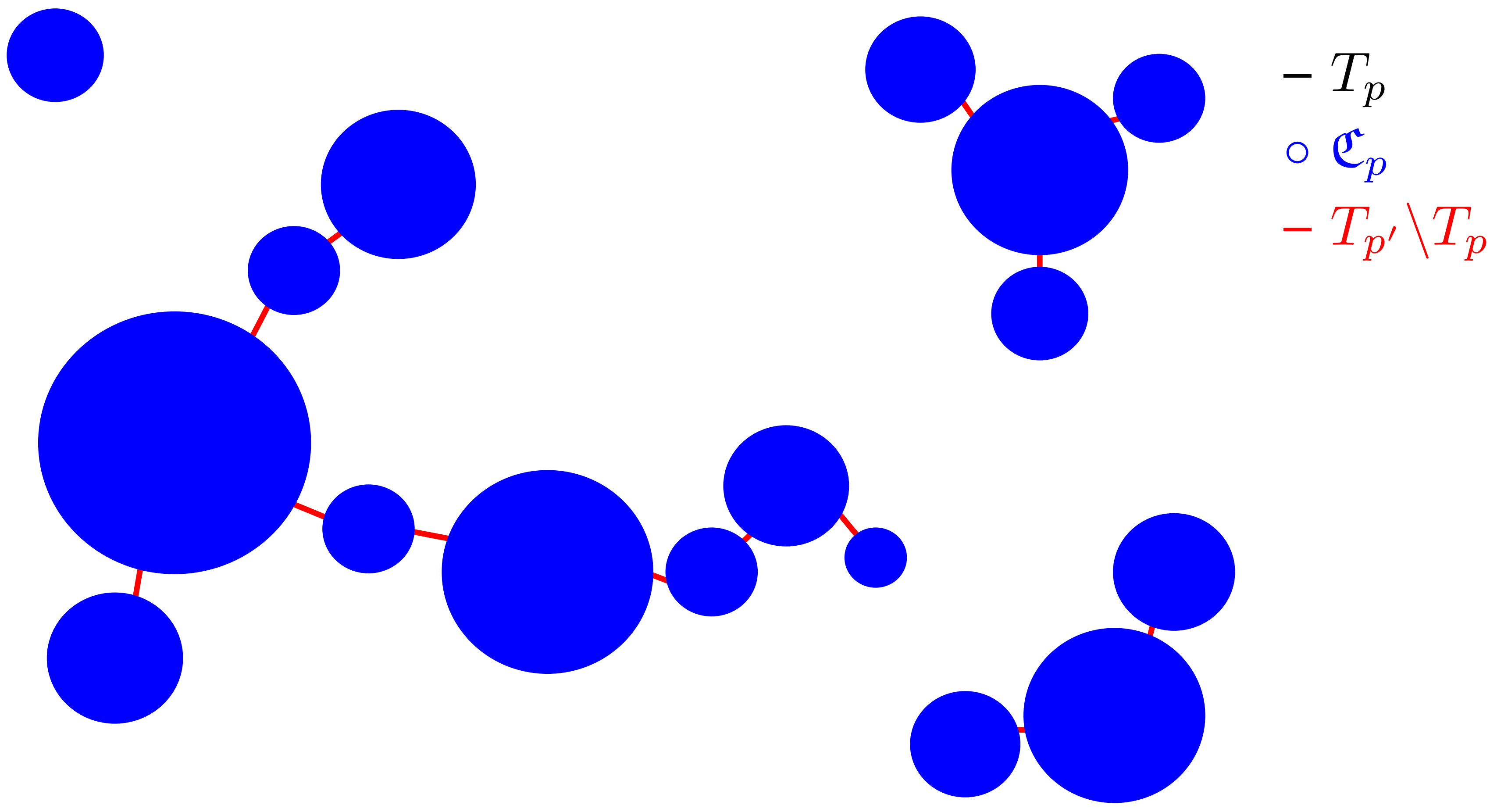

Approximation with $\times$-graphs

Proposition

$p, p' \approx$ critical : conditionally to H_p, $H_{p,p'} \approx \times$-graph.

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- $\forall a \in I$ weight $w_a \in \mathbb{R}^+$. $q \in \mathbb{R}^+$.
- Indep, $\forall i, j \in I$: $\mathbb{P}((i,j) \in G_p) = 1 - e^{-w_i w_j q}$.

- weight$(C_i) =$ #vertices$|C_i|$.
- $\forall A, B$ clusters, let $\Delta_{A,B} := \#\{$edges between $A, B\}$.

$$\mathbb{P}((A,B) \in H_{p,p'} | H_p) = 1 - ((1-p')/(1-p))^{\Delta_{A,B}}.$$

$$\Delta_{A,B} \approx\propto |A||B|?$$

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Heuristic: Large clusters $\approx$ sets of i.i.d. uniform vertices.

Diameter $\lesssim (n^d)^{1/3} \gg n^2$ mixing time on $(\mathbb{Z}/n\mathbb{Z})^d$.

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Need limit of $G_\times$: Equiv $\sigma_2 := \sum |A|^2$, $\sigma_3 := \sum |A|^3$. Upper-b $\max |A|$.

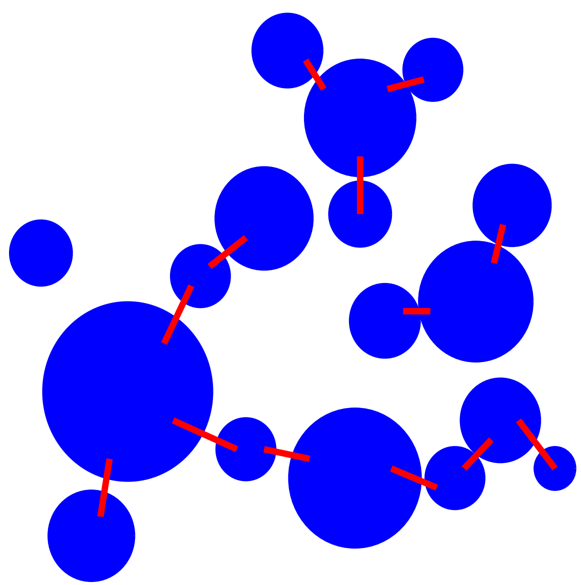
- $\sigma_2 \asymp (p_c - p_s)^{-1}$; $\sigma_3 \asymp (p_c - p_s)^{-3}$
- σ_2 and σ_3 concentrated

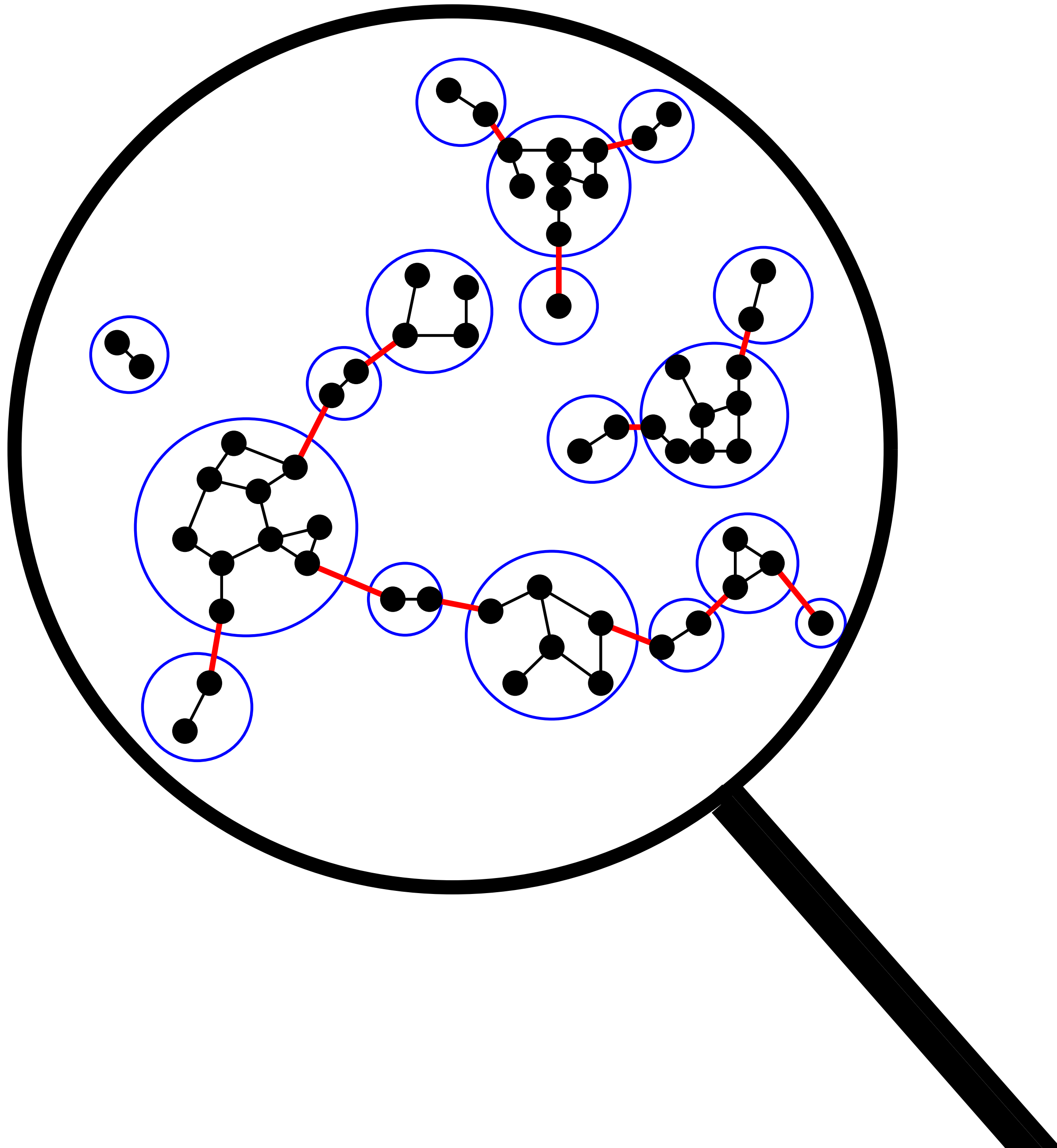
Need coupling :

$$\sum (\Delta_{A,B} - \circledast |A||B|)^2 \ll \sigma_2^2.$$

Geometry in the torus

Cluster graph $\approx \times$ $\approx$ Critical torus





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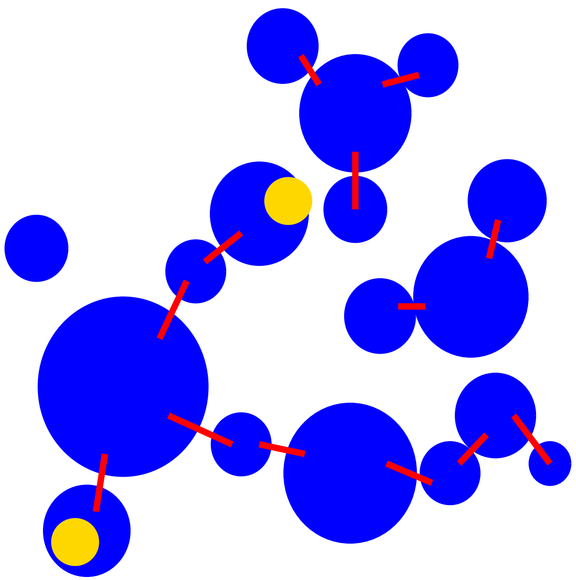

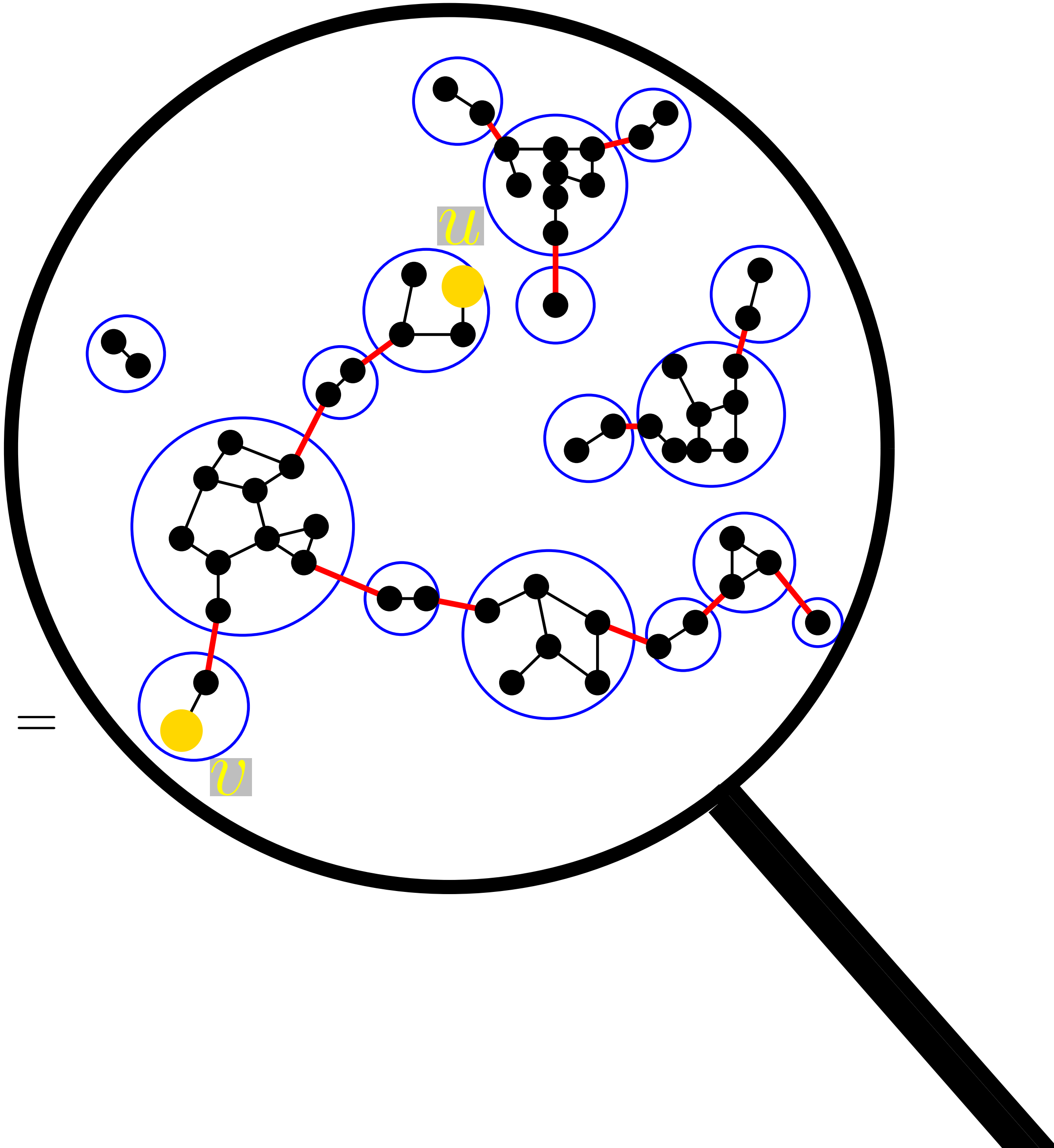

Bad method: $d(u,v) = \sum[\dots] =$
$1+1+4+1+1+1+2$

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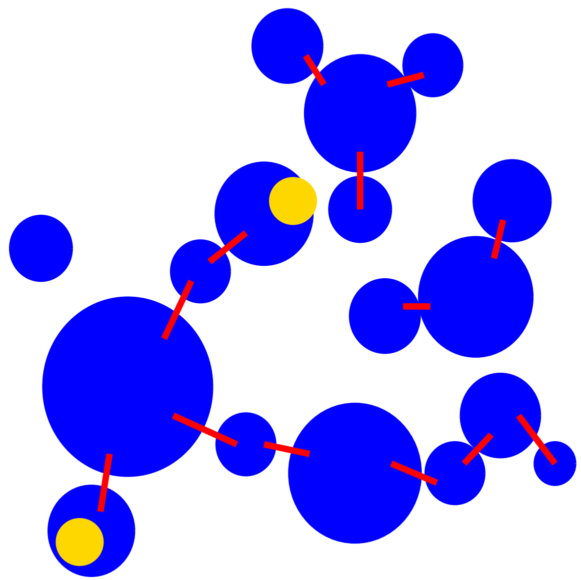
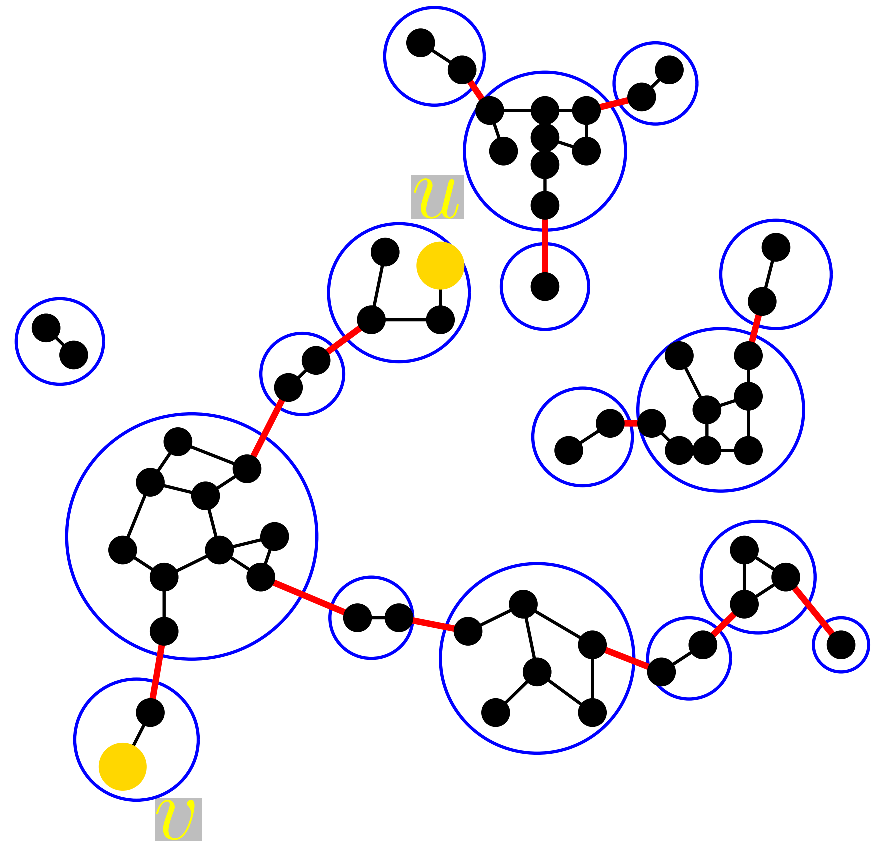


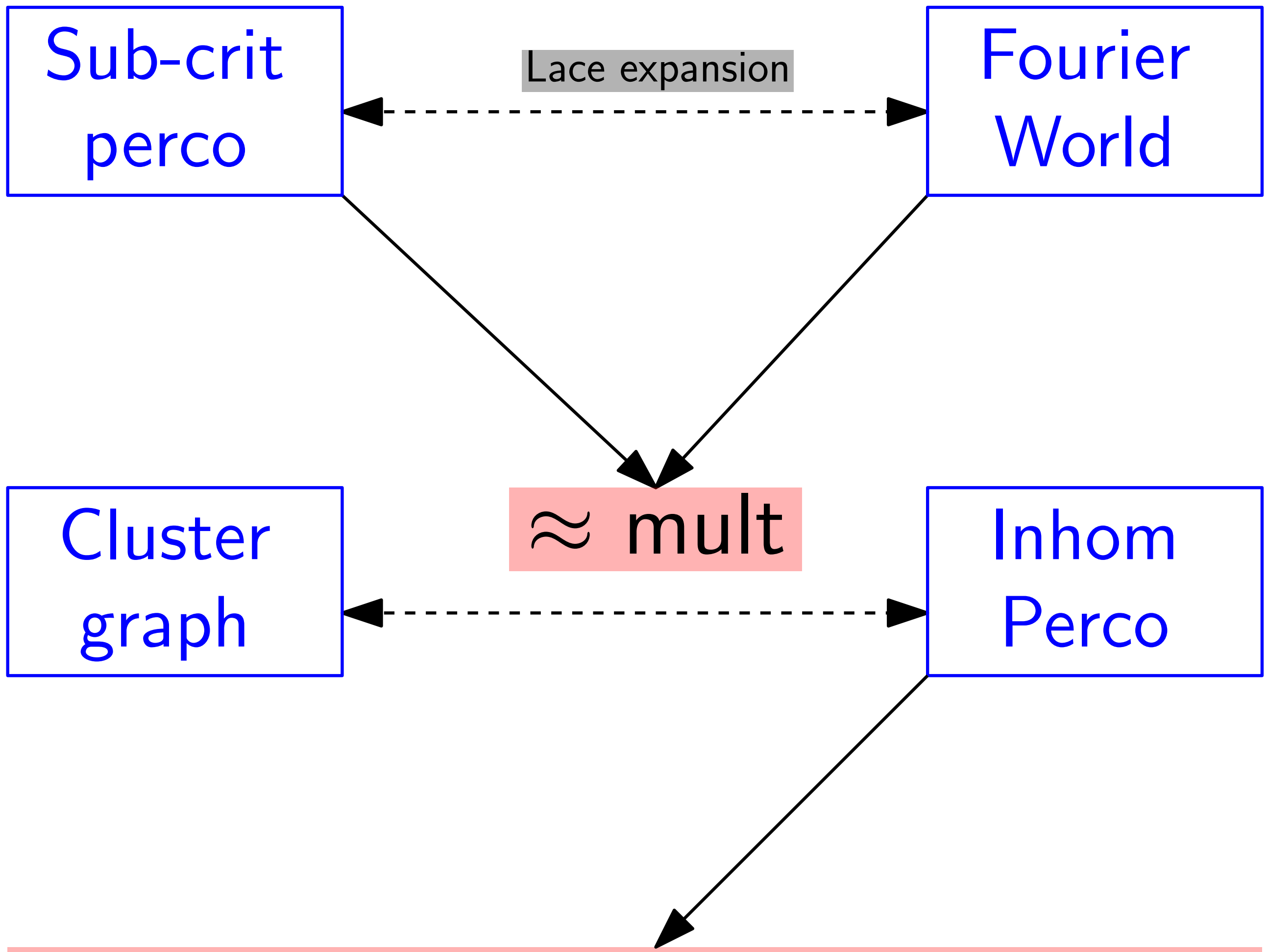


Bad method: $d(u,v) = \sum[\dots] = 1+1+4+1+1+1+2$

Good method :
$$d_{\text{piv}}(u,v) \approx \frac{p_c}{p_c - p_s} d_{\text{clust}}(u,v).$$

- Critical torus $\approx$ forest $\implies$ $\approx$ 1 single path γ between u and v
- $\#$ closed pivotal edges of γ in T_p $\approx (p_c - p_s)/p_c|\gamma|$ and $\approx d_{\text{clust}}(u,v)$.

II Proof of $\Delta_{A,B} \approx \circledast |A||B|$.

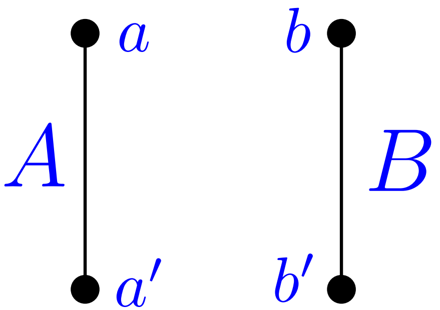


$$\sum_{A \neq B : \text{grands}} (\Delta_{A,B} - \circledast |A||B|)^2 \ll \sigma_2^2.$$

Splitting the sum

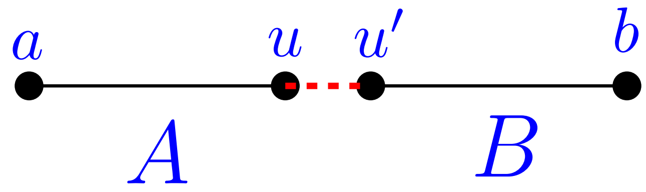
Upper-bound

$$\mathbb{E}[\sum |A|^2|B|^2]$$



Lower-bound

$$\mathbb{E}[\sum \Delta_{A,B}|A||B|]$$



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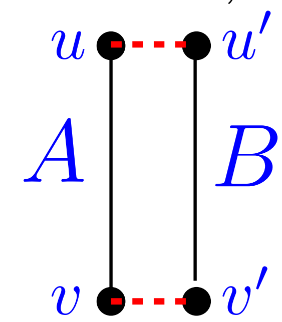

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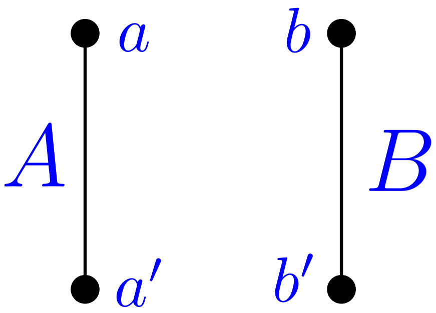
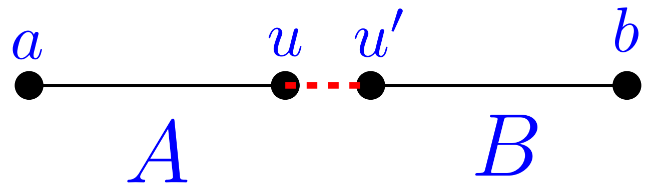


Classical (BK)

Lower-bound

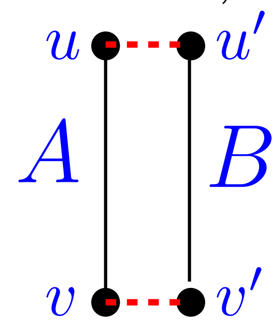
$\mathbb{E}[\sum \Delta_{A,B}|A||B|]$



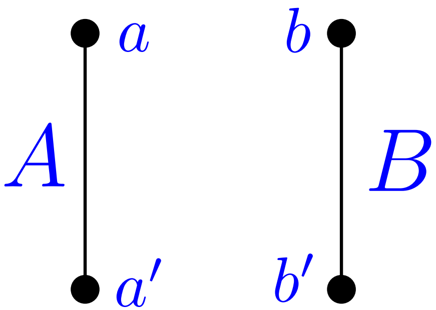
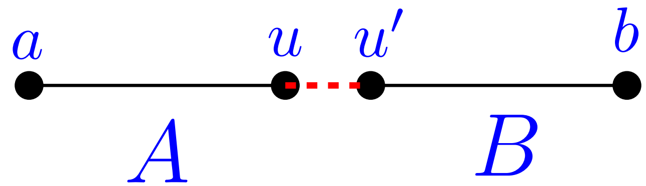
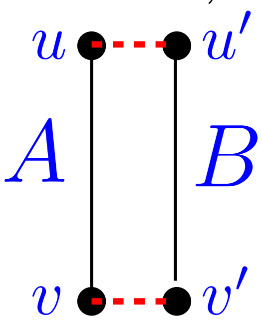
Classical
(Off-method+Triangle)

Upper-bound

$\mathbb{E}[\sum \Delta_{A,B}^2]$



Splitting the sum

Upper-bound	Lower-bound	Upper-bound
$\mathbb{E}[\sum \lvert A\rvert^2 \lvert B\rvert^2]$	$\mathbb{E}[\sum \Delta_{A,B} \lvert A\rvert \lvert B\rvert]$	$\mathbb{E}[\sum \Delta_{A,B}^2]$
		
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Splitting the sum

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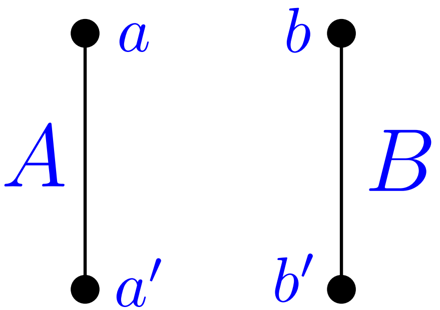	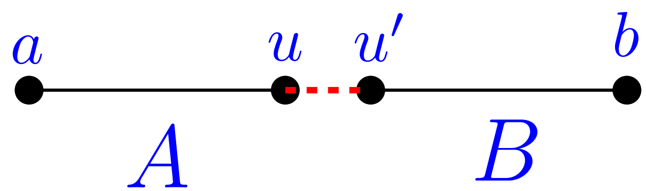	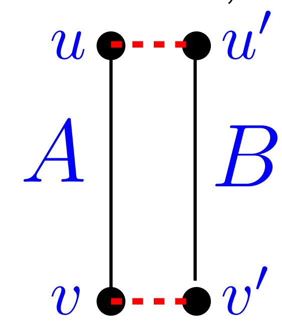
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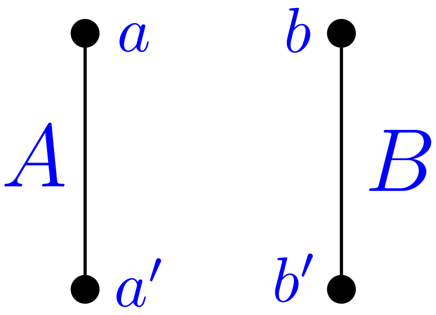	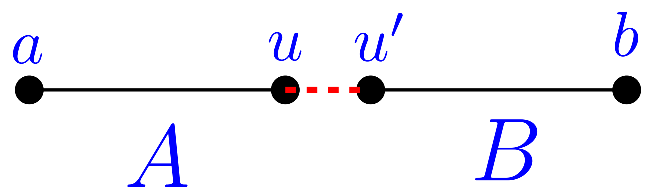	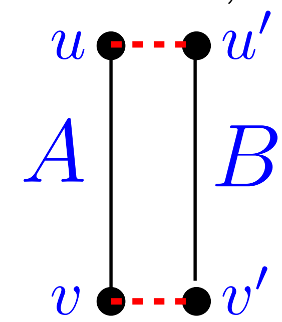
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Weak plateau

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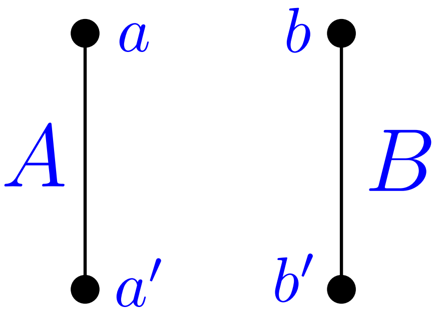	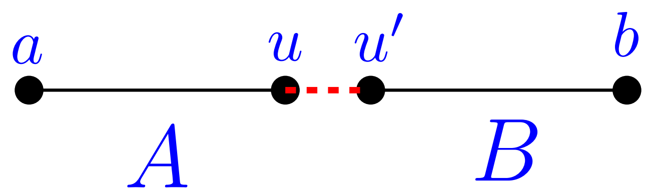	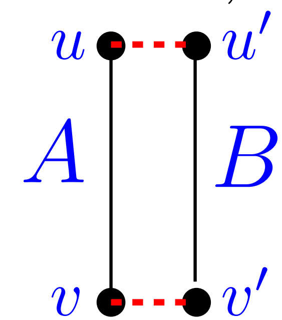
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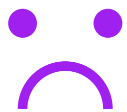

Graph $\approx \times$:

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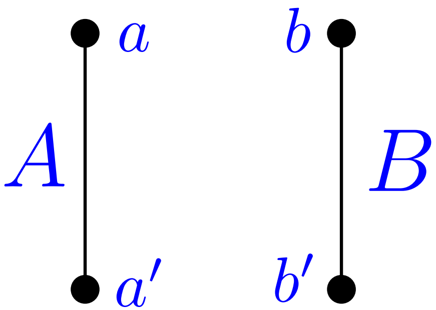	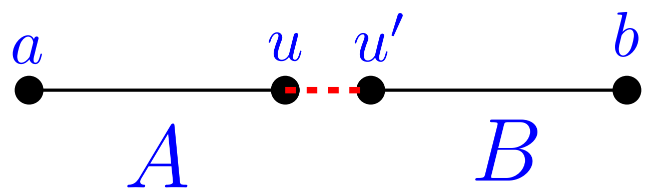	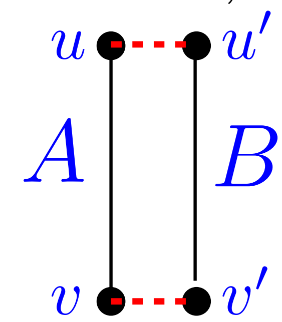
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Graph $\approx \times$:

$\hookrightarrow$ Cluster graph

Inhomogeneous percolation : plateau

Vertices I. Probas $P = (p_{i,j})_{i,j \in I}$. Indep $\mathbb{P}((i,j)$ open$) = p_{i,j}$.

Study: $T = (\mathbb{P}(i \leftrightarrow j))_{i,j \in I}$

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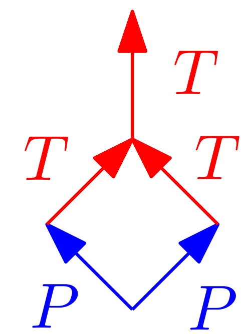
Lower-bound : Inclusion-exclusion.

Exclusions : 2 paths $\hookrightarrow \in$ cycle $\hookrightarrow$ very costly.



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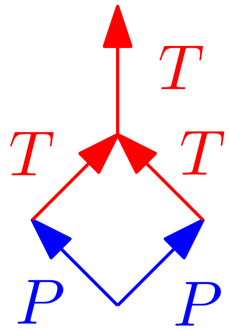

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We need:

- Few cycles $\hookrightarrow$ local cycles disapear
- $\|P\|_2$ small $\hookrightarrow$ Weak plateau
- t_{mix} small. $\hookrightarrow$ Spectral gap of $(\Delta_{A,B})_{A,B \in \mathfrak{C}}$.

Spectral gap

Look at: $\lambda_1, \lambda_2, \ldots$ eigenvalue of $(\Delta_{A,B})$

Lower-b λ_1: $\lambda_1 = \max \frac{\langle \Delta X, X \rangle}{\|X\|^2} \hookrightarrow X := (|A|)_{A \in \mathfrak{C}}$. (already done)

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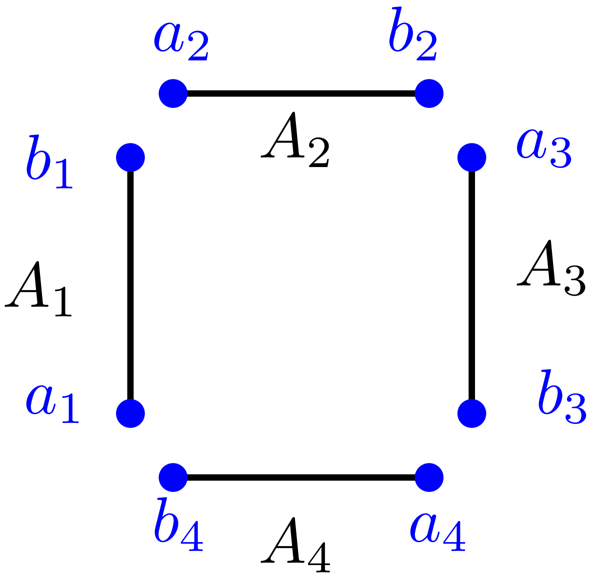

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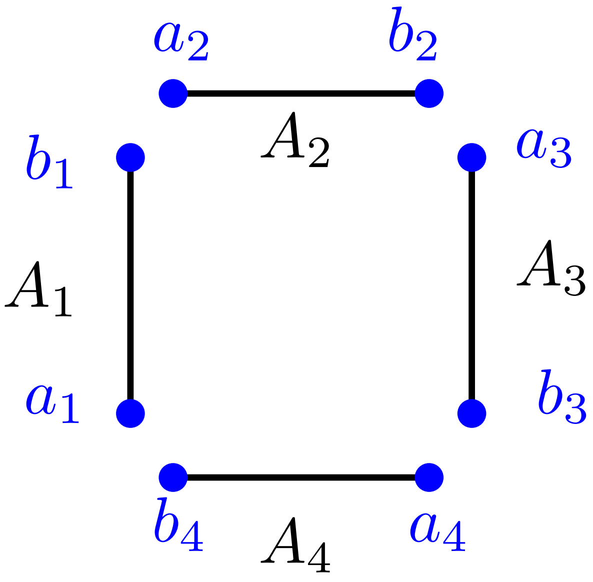

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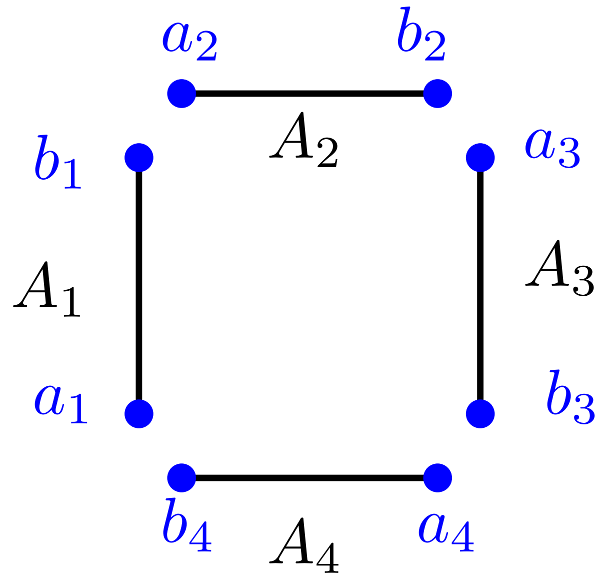

Welcome in FOURIER's magical world....

$$\sum_{x \in \text{dual tore}} \hat{\tau}(x)^k \hat{D}^k(x)$$

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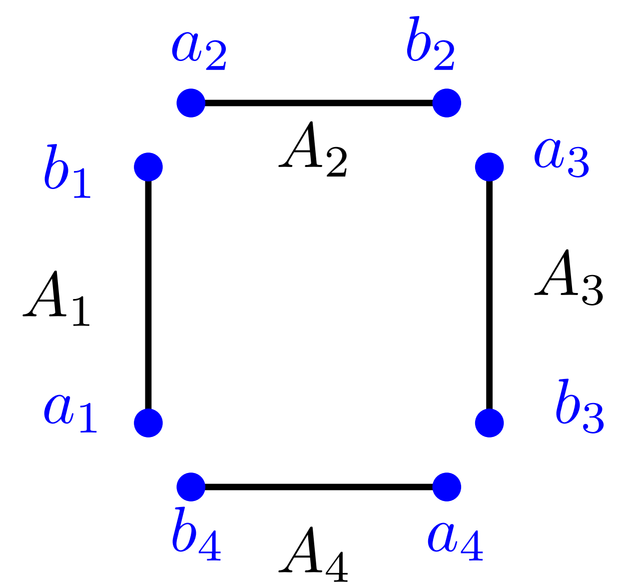
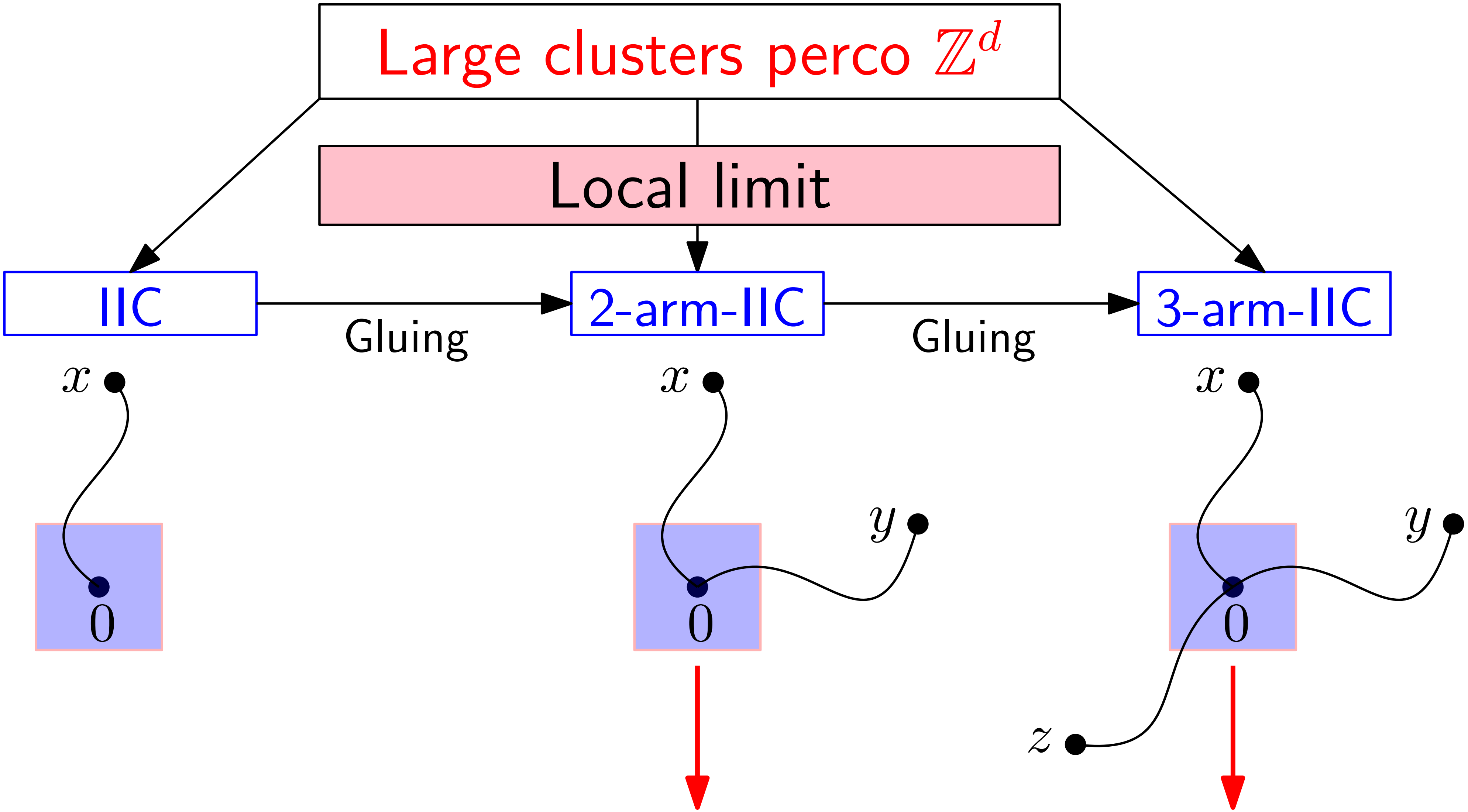


Welcome in FOURIER's magical world....

$$\sum_{x \in \text{dual tore}} \hat{\tau}(x)^k \hat{D}^k(x) \hookrightarrow$$ Infrared-bound ($\approx$ Fourier's plateau).

Ok for $k \geq 4$.

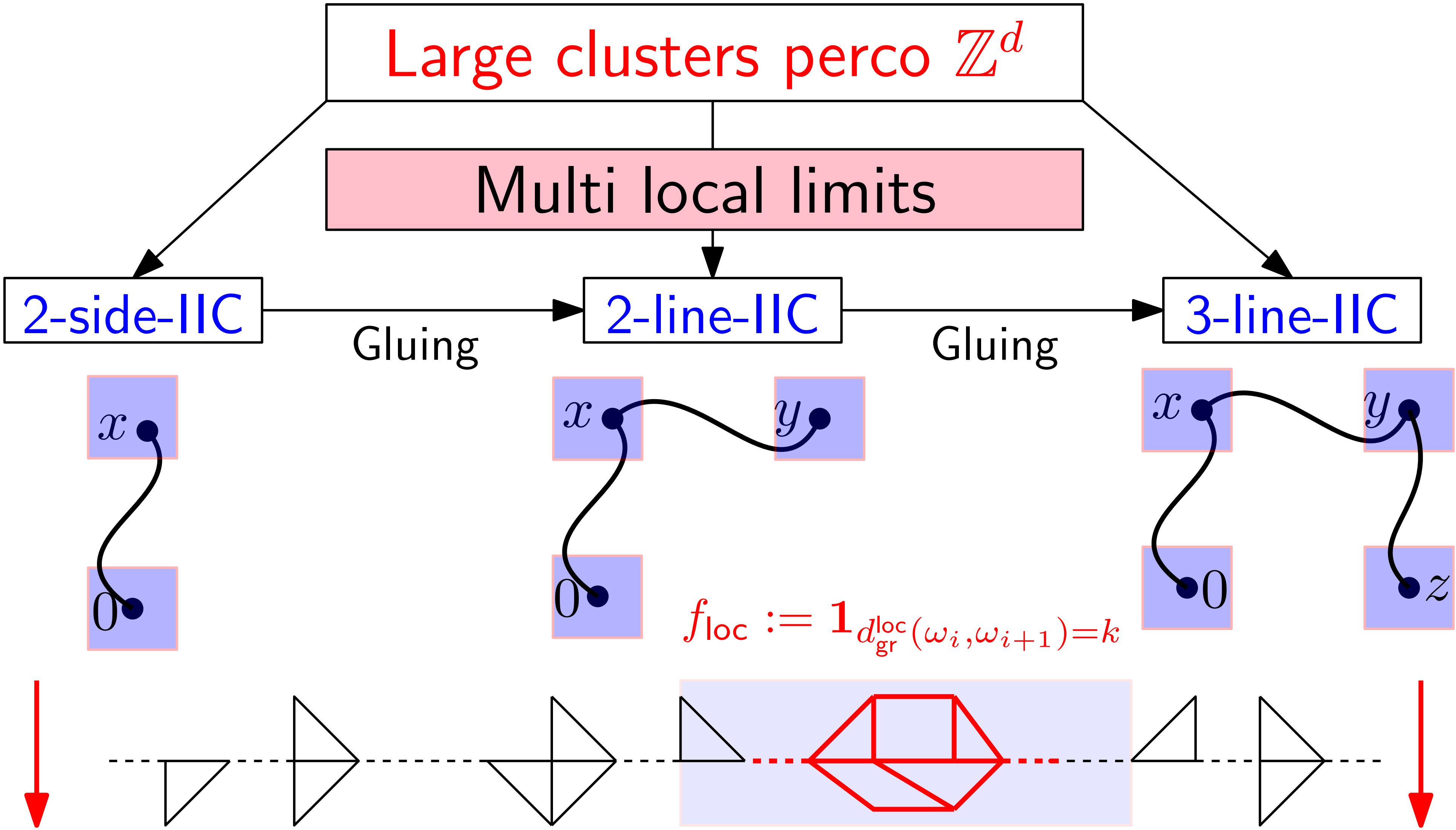
III Equivalent susceptibilities



Proposition

$$\chi(p) := \mathbb{E}_p[|C(0)|] \sim \frac{C_\chi}{p_c - p} \quad ; \quad \mathbb{E}_p[|C(0)|^2] \sim C_\Psi \chi(p)^3$$

IV $d_{\mathsf{gr}} \sim C d_{\mathsf{piv}}$.



Proposition

$$\frac{1}{\chi(p)} \sum_{x \in \mathbb{Z}^d} \mathbb{P}(0 \leftrightarrow x), |d_{\mathsf{gr}}(0,x) - C d_{\mathsf{piv}}(0,x)| > \epsilon \chi(p)) \longrightarrow 0.$$

High level summary: the contraction

Random walk bounds at $p < p_c$ for $\mathbb{P}_p(0 \leftrightarrow x)$

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Correlation
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And (large) cycles
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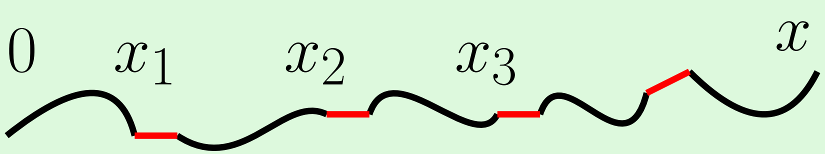
Paths at $p' > p$



$\approx$ many $\approx$ inde. p-open paths

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Convolutions contract toward Gaussian

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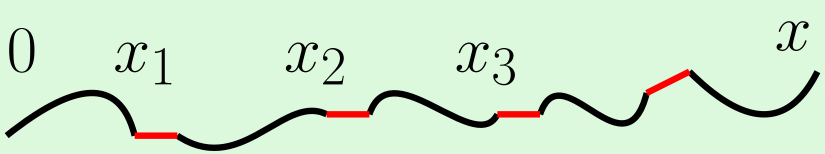

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Better RW bounds for :

$$\sum \mathbb{P}_p(0 \leftrightarrow x_1)\mathbb{P}_p(x_1 \leftrightarrow x_2)\ldots$$

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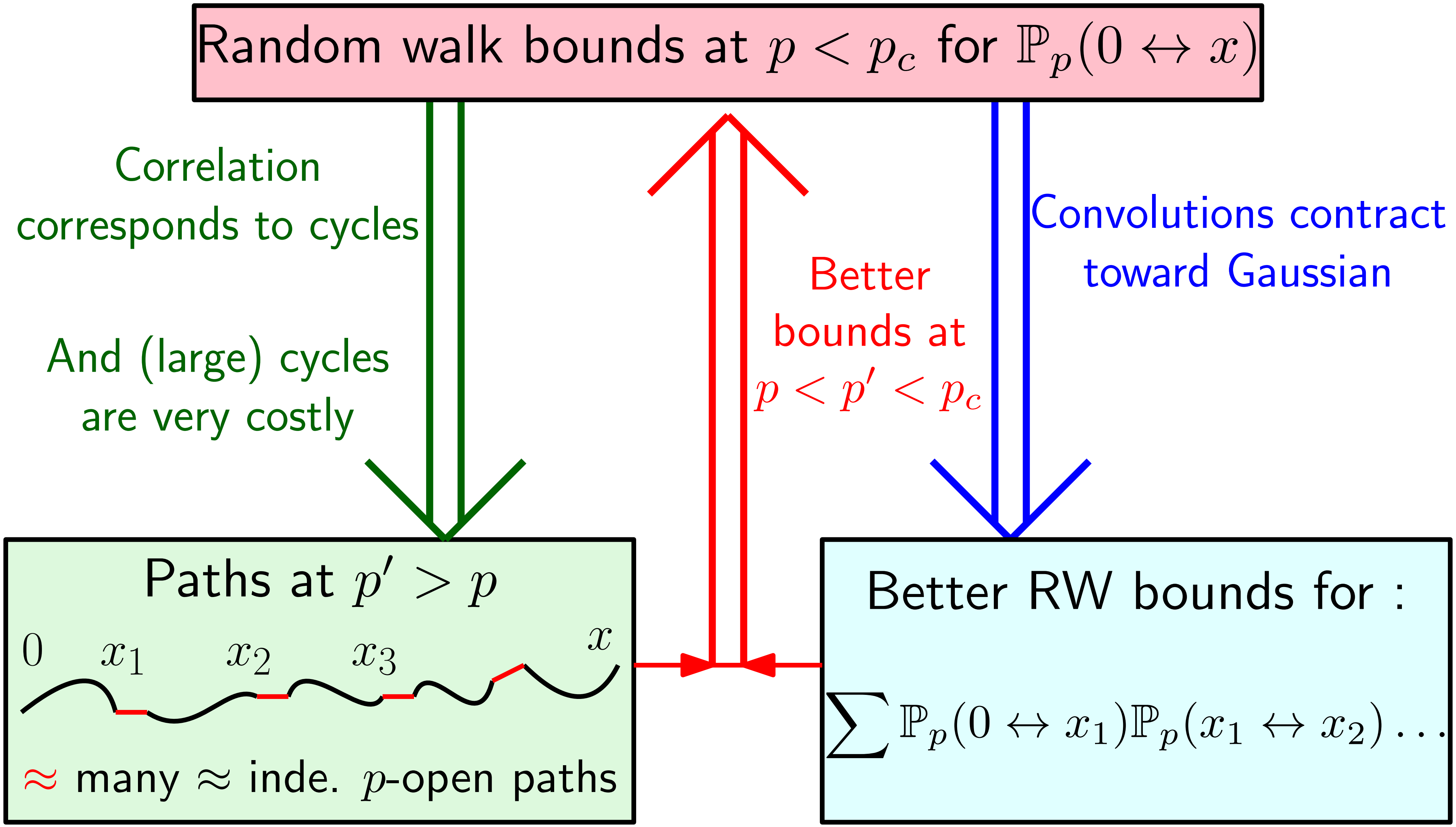

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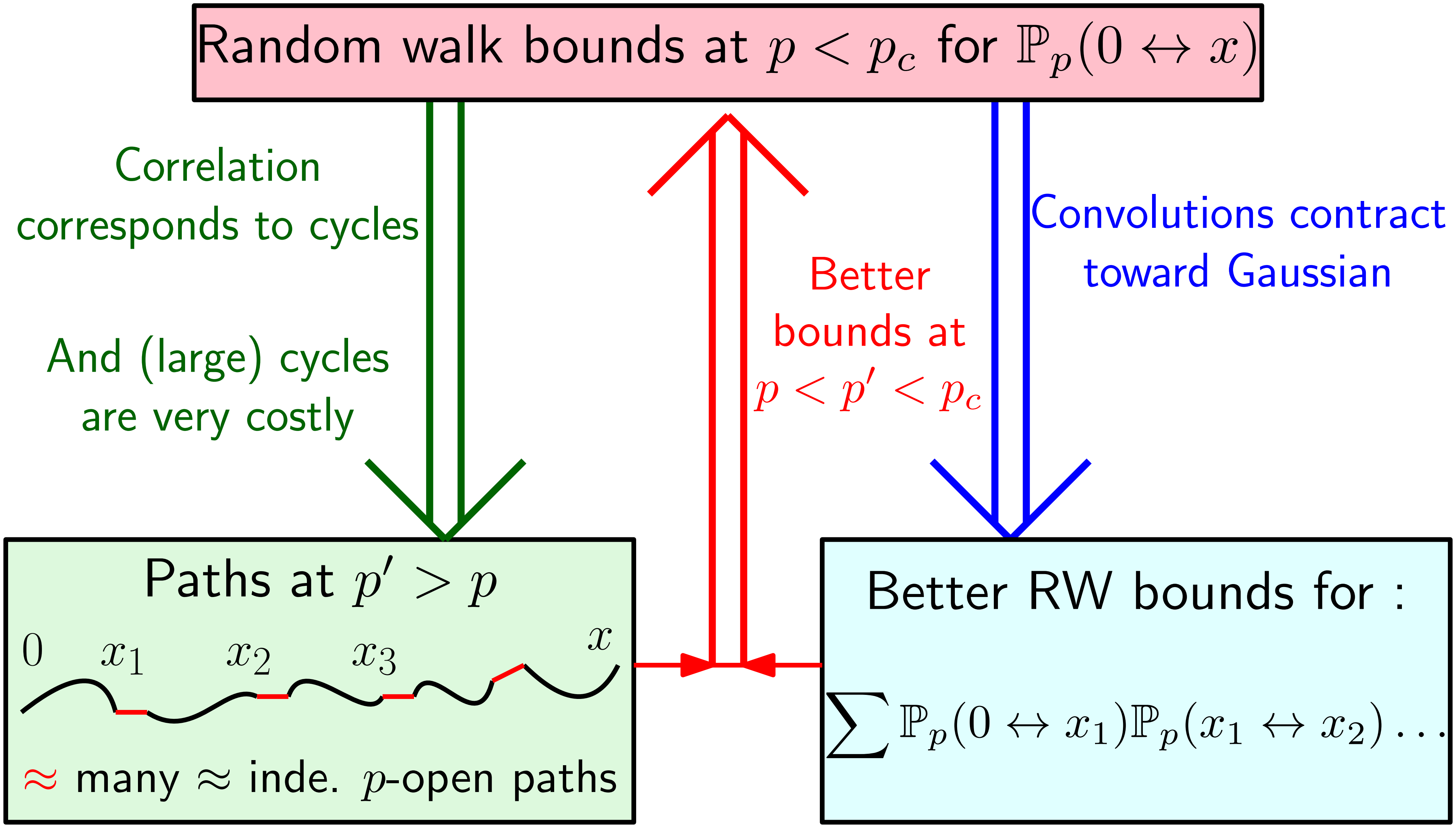
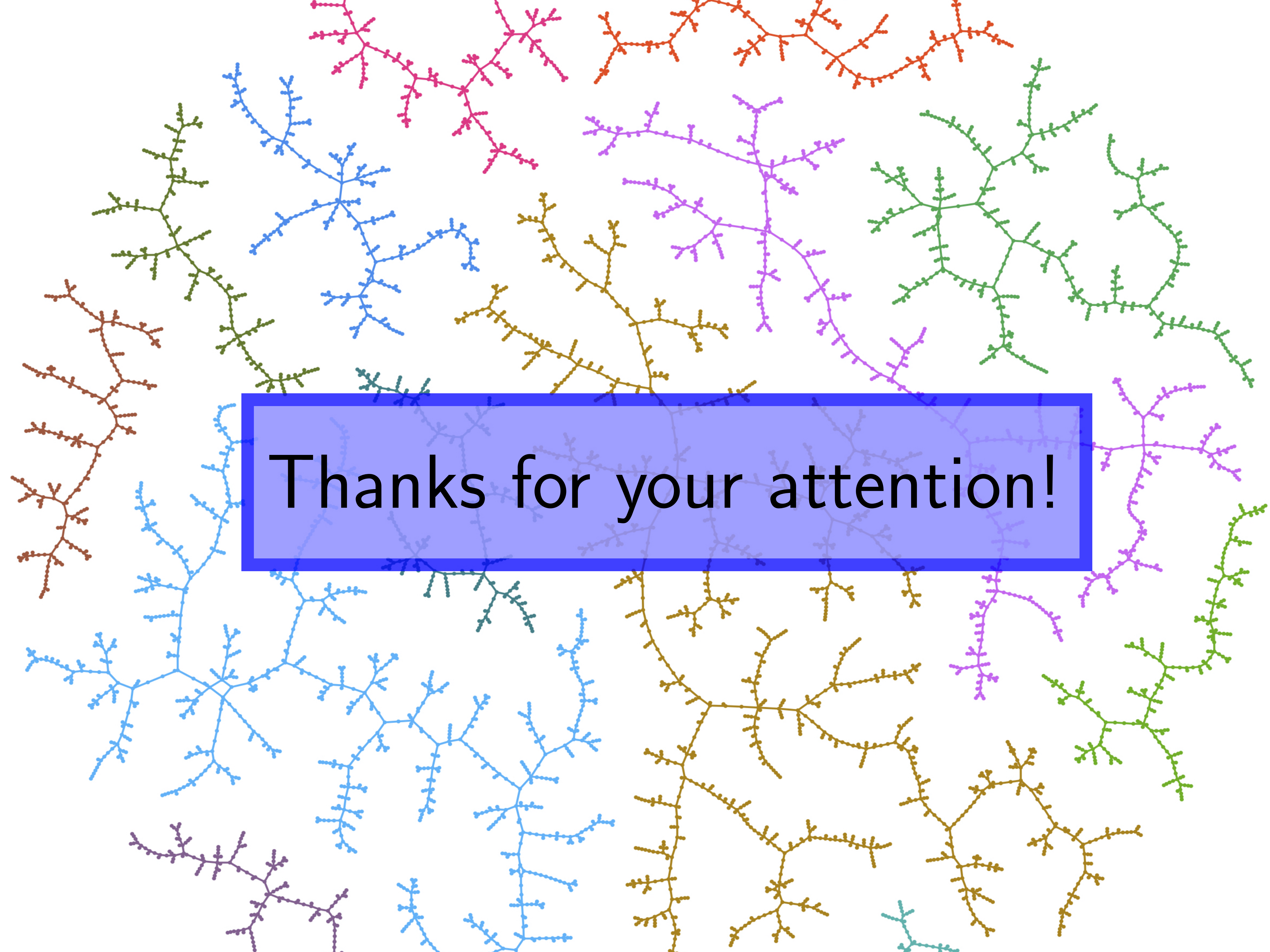


Ongoing : New proof for $\theta(p_c) = 0$ for $d > 6$!

Paths at $p' \approx$ Markov chain of p-open connected components


Thanks for your attention!